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Progress in Physics (57)

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Metamaterials are smart materials whose properties arise from a clever engineering of their structure rather than from their composition. This fairly broad definition encompasses materials such as semiconductor hetero-structures for the investigation of low-temperature quantum phases, materials with tailored electromagnetic properties such as Faraday cages, photonic crystals employed in telecom applications, or mechanical metamaterials used for vibration isolation or acoustic cloaking. Despite the fact that in the above examples length- and temperature-scales can range from the nanometer to architectural scales, and from sub-Kelvin to room temperature, such metamaterials often follow common design principles that lead to their specific functionalities. Recently, such a new design principle has been established for photonic and mechanical metamaterials: Motivated by the success of the use of a mathematical concept called "topology" in the description of quantum-mechanical low-temperature phases of electrons, classical topological mechanical and photonic metamaterials have been suggested and materialized.

Topology is a mathematical concept that describes entities in terms of properties that do not change under a smooth deformation of the objects under investigation. Let us translate this to the engineering world: If a targeted functionality can be bound to such a topological property of the metamaterial, it will be realized with near perfection despite the potentially substantial production imperfections. Given this "failsafe" approach, it is not surprising that the emerging and still rather young field of topological metamaterials is gaining ever more momentum.

In our work we focus on topological effects in mechanical systems [1,2]. In particular, we try to make use of one of the core features of topological systems: the existence of stable surface modes whose presence is dictated by the structure of the bulk. This so-called bulk-edge correspondence gives rise to the "failsafe" design principle. As long as imperfections in the system do not change the bulk properties in a fundamental way, the surface will have intriguing properties such as the presence of unidirectional wave-guides for lattice vibrations.

When discussing topological mechanical metamaterials, we can group them into three broad classes [3]. First, there is an important difference between topological features around zero frequency and those at high, i.e., non-zero, frequency. The former have direct implications on static and thermodynamic properties of the metamaterial, while the latter are useful for applications where the transport of mechanical energy in the form of lattice vibrations plays a role. For the high-frequency applications, we can make a further distinction between so-called active metamaterials, where energy has to be injected from the outside and passive materials that possess topological properties also in the absence of an external drive. In this progress article we will highlight the advances in the field over the last few years both on zero-frequency phenomena as well as on active and passive metamaterials that capitalize on topological effects at high frequencies.

Zero frequency properties: Maxwell frames

Mechanical systems are described by a second order differential equation, Newton's equations of motion. For a generic system, all coefficients appearing in this equation describe coupling strengths of different local modes, effective mass densities or externally applied forces. All of these result in real coefficients, which in turn imply certain symmetries of the equations. In particular, for a collection of masses interacting via central forces, the equations are formally identical to particle-hole symmetric electron systems known from the quantum theory of superconductivity [4]. It is this symmetry analogous to the quantum mechanical particle-hole symmetry which enables the topological description of zero-frequency properties of a special class of mechanical systems known as Maxwell frames.

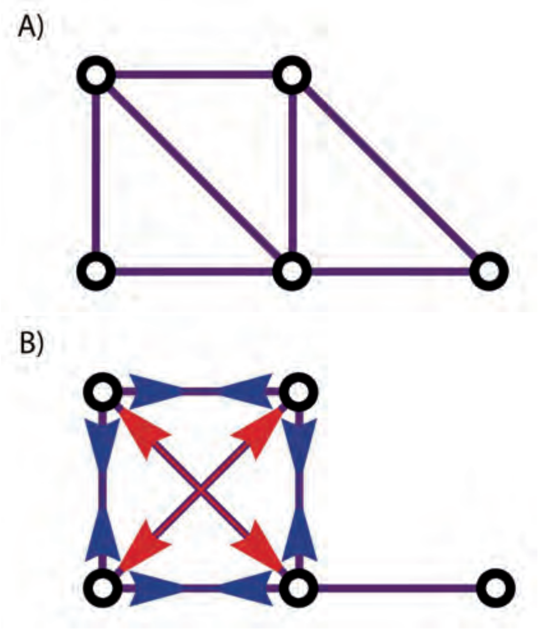

Figure 1: A) A finite Maxwell frame with no zero modes. B) A Maxwell frame where the rightmost point can be rotated freely at the expense of a state of self stress: If the bars are stressed like indicated by the arrows, no net forces act on the points.

A general frame is a system of (stiff) rods connected by perfect hinges, cf. Figure 1a. A Maxwell frame is such a collection of rods and hinges where the number of degrees of freedom of the hinges exactly matches the number of constraints imposed by the rods. Owing to this perfect balance, one could expect that there are no freely movable parts, i.e., zero modes, in such a frame. However, a simple counting argument due to Maxwell and Calladine [5] makes the situation more transparent:

#(zero modes) – #(states of self stress)
= #(degrees of freedom) – #constraints.

Clearly, even in a Maxwell frame, where the right-hand side vanishes, there can be zero modes if rods are "wasted" between hinges that would otherwise already be full constrained. These redundant rods give rise to states of self-stress, where there can exist stresses on the rods without net forces on the hinges, cf. Figure 1b.

The topological description of periodic Maxwell frames by Kane and Lubensky [4] is making predictions of how these zero modes and states of self-stress are distributed in an otherwise balanced frame. In particular, if a periodic Maxwell frame is cut open to a finite system, one necessarily obtains zero modes. The topological polarization $\mathbf{R}_T$ of Kane and Lubensky is indicating where on the surface these zero-modes appear [4].

The topologically polarization $\mathbf{R}_T$ is a topological quantum number intrinsically bound to one-dimensional systems. This restriction arises from the fact that the polarization $\mathbf{R}_T$ is nothing but the winding number known for electron systems in class BDI in the Altland-Zirnbauer classification [6]. This class is known to only host topological phases in one spatial dimension.

These topological Maxwell frames have been materialized in various experiments over the last few years [7-10]. Zero-modes of a one dimensional structure have been shown to exist in a toy system made from LEGO [10]. Moreover, it was observed that these zero modes seed a non-linear conduction mechanism.

There is, however, a way to get higher dimensional systems described by the Kane-Lubensky theory. By stacking one dimensional systems to two- or three-dimensional lattices, the polarization $\mathbf{R}_T$ can be carried over to higher dimensions. A procedure known from electronic systems in the framework of "weak" topological indices. Indeed such stackings have been proposed [11] and offer interesting future directions for the design of anisotropic elastic materials.

Topological surface states around zero frequency are obviously relevant for the thermodynamic low-energy physics of a mechanical system. However, the constraint on Maxwell frames makes these states somewhat non-generic. Moreover, one could ask if phononic transport at explicitly non-zero frequency could enjoy topological protection.

High frequency properties: Wave guiding

High-frequency topological band-structures are not bound to Maxwell frames but can in principle be found in any mechanical structure. However, in this more general case the topological classification is more intriguing. In a recent publication [3] we could show how the "periodic table of topological insulators" by Kitaev [12] and Ryu and collaborators [13] translates to the mechanical world.

The periodic table of topological insulators offers a look-up table if for a given symmetry and spatial dimension a system can harbor a topological band structure. When translating this table to classical systems it is important to note that certain topological phenomena, such as the quantum spin Hall effect for electrons observed in HgTe-CdTe quantum wells [14], requires a time-reversal symmetry T that square to minus one $T^2 = -1$. This behavior is known from quantum spin-1/2 objects like electrons and it is not a priori clear if the same physics can be observed in a classical system.

In our recent experiment we have answered this question affirmatively. We could show how a collection of pendula, coupled via springs, lever arms, and ball bearings, can display all features of topologically protected edge modes. In particular, we have shown that the spectrum of surfaces states is given by two counter-propagating phonon modes that differ by an internal degree of freedom akin the two spin states of electrons in HgTe systems, cf. Figure 2. Moreover, we demonstrated that the surface phonons are immune to a broad class of disorder, fulfilling the promise of offering a design principle for metamaterials which are immune to production imperfections.

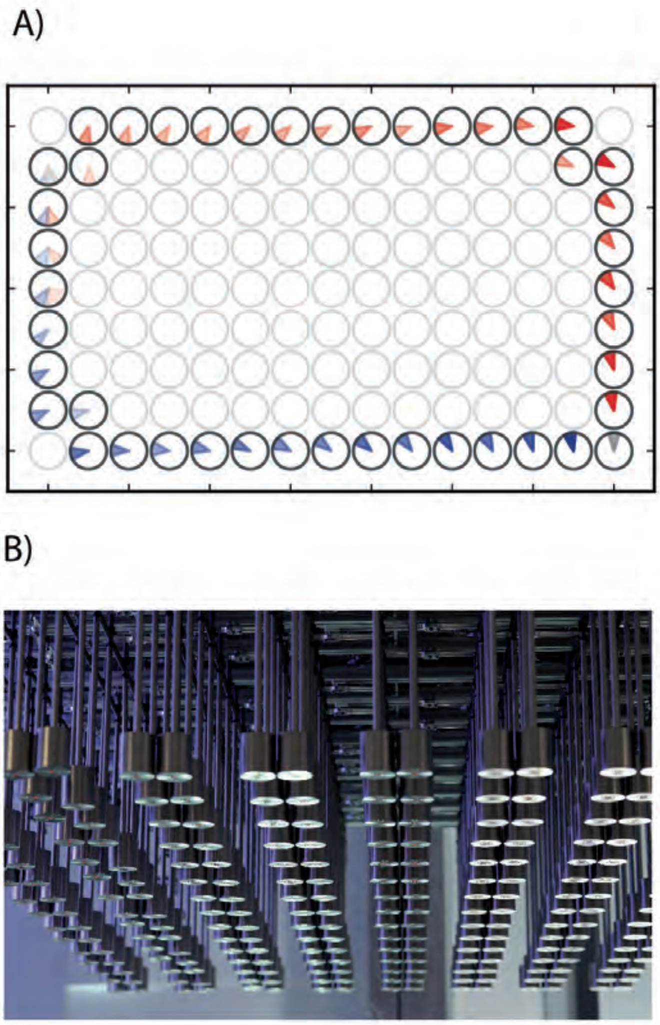

Figure 2: A) Observation of helical phonon transport. Each site represented by a circle hosts an effectively two-dimensional pendulum. The wedge indicates when the wave packet injected in the bottom right circle was measured at the respective site, with time running counter-clock wise. The gray color in the bottom right site represents a linearly polarized wave packet which splits into a left- (red) and right- (blue) circularly polarized wave packet, each running around the sample in opposite direction. This locking of the polarization to the propagation direction is the hallmark of the helical edge channels in the quantum spin Hall effect. B) Picture of the experimental setup.

It is worth mentioning that beside our setup, which relies on a specific symmetry and offers the phenomenology of helical (bi-directional channels differing by their "spin") rather than chiral (strictly uni-directional) channels, there have been experiments demonstrating the latter [15]. In a collection of magnetically coupled gyroscopes, Nash and co-workers could demonstrate acoustic surface states that are strictly chiral akin the phenomenology of the quantum Hall effect discovered in the early 80ies of the last century [16]. Moreover, their system does not require any symmetry and is hence extremely stable against any type of disorder.

What did we achieve by exploring topological band structures beyond the quantum mechanical world of electrons? The phenomenology and theoretical description of these electronic systems is well understood and there is little hope that the mechanical models add significantly to our understanding of such electron systems. However, the fabrication and measurement techniques in the classical mechanical world are significantly simpler and under much higher control than their electronic counterparts. This in turn enables us to explore new devices in an efficient way. Moreover, for technological applications the ease of fabricating at the millimeter or micron scale rather than at the nano-scale might lead to industrial application of devices based on topological phenomena in a comparatively short time.

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