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A golden age for black hole science

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Black holes, once thought to be the physically irrelevant figments of mere mathematical speculation, have spectacularly risen to the forefront of physics research, anchored in the realm of physical reality by ingenious astronomical observations, visualized through innovative and computationally intensive imaging techniques, and propelled by entirely new observational signatures carried by gravitational waves. At the same time our theoretical understanding of black holes keeps advancing, opening new vistas on their mysterious properties. Taken together, these advances herald the advent of a golden age of black hole science.

What is a Black Hole?

Black holes have long been objects of scientific imagination, at first confined to the realm of prescient theoretical speculation [1], which was met with intense skepticism [2], epitomized in a quote of Arthur Eddington, [3],

> *"I think there should be a law of Nature to prevent a star from behaving in this absurd way."*

Inspite of this early opposition, over time they came to be recognized as inevitable consequences of the laws of gravity and inextricable parts of reality. General Relativity (GR) harbors within it the robust prediction [4] of the existence of localised massive astrophysical objects whose behavior is dramatically different to that of stars or planets. Black holes are characterised by their event horizon, an imaginary surface demarcating a region in space and time from which nothing, not even light can escape. Yet, in accordance with the equivalence principle, this surfaces carries no perceptible material qualities, as would for example the surface of a star. It is in fact impossible to detect an event horizon using local classical measurements. Bizarre and exotic as they may seem, the idea of such objects predates Einstein and GR by over 130 years. Both Michell [5] and Laplace [6], who subscribed to the corpuscular theory of light, pointed out that dense enough objects, even when treated by the Newtonian methods of the time, would eventually posses an escape velocity exceeding that of the speed of light, effectively trapping its corpuscles inside their immense gravitational potential. Their analyses even allowed them to determine the radius of a such a "dark star". In modern notation, their result corresponds to [1]

$$R_s = \frac{2G_N M}{c^2}, \qquad (1)$$

in terms of the mass M of the hole, the Newton constant, G_N, and the speed of light, c. So enormous is the density of these objects that a black hole weighing in at one Terrestial mass would be compressed to a Schwarzschild radius of around 9 mm. Owing to their light-trapping property, they would be unobservable in optical telescopes, or indeed any other instruments relying on electromagnetic messengers, except, as Michell mused, by observing the effect of their

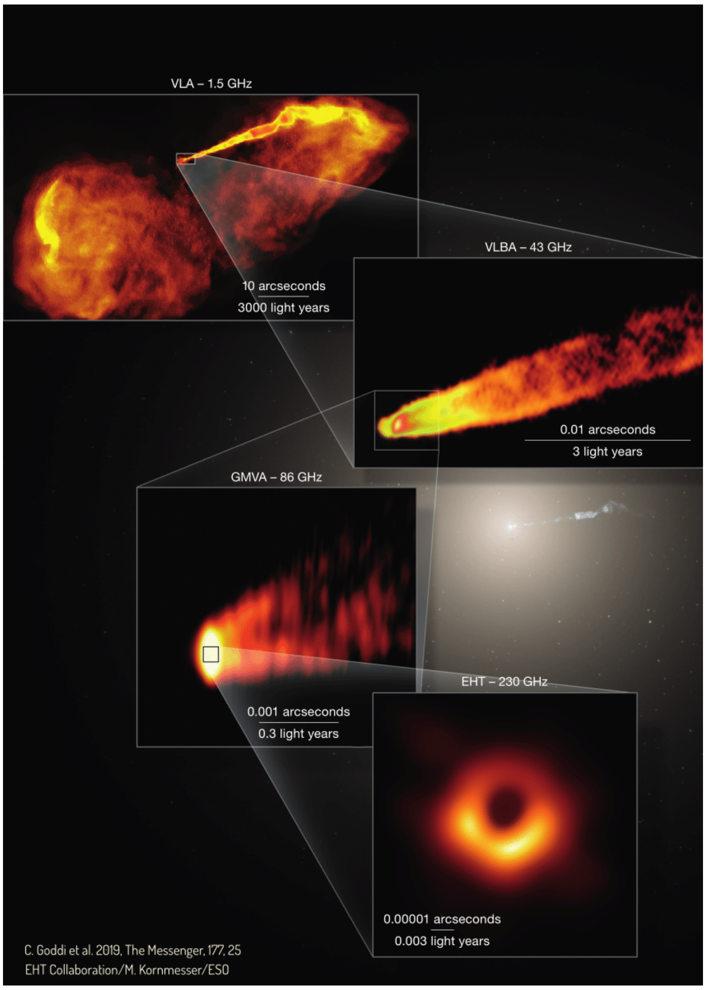

Fig. 1: Turbulent environs of the black hole M87. At the scale of 3 kly we see synchrotron emission emanating from the powerful jet sourced by the hole. Zooming into the source and panning through the EM spectrum eventually reveals the shadow of the black hole M87. See also Fig. 2 [Image Credit: EHT Collaboration].

gravitational field on their surroundings. This feat was eventually accomplished by two teams of observational astronomers led by Genzel, [7] and Ghez [8] around the dusk of the second millennium, who through a careful study of the orbital parameters of nearby stars, inferred the existence of a supermassive black hole in the center of our own Milkyway galaxy, located in the sky within the configuration Sagittarius (Sgr A*). The inferred mass of this black hole is estimated to be some $4.3 \times 10^6\, M_\odot$. These observations give indirect, yet incontrovertible evidence that black holes exist in our Universe. Far from being the quiet dark objects suggested by their name, black holes are powerful sources of radiation across the electromagnetic spectrum, which opens up the possibility of more direct observational approaches.

Seeing The Invisible

While black holes are arguably the simplest, most pristine solutions of the Einstein equations imaginable, they source immense gravitational forces and cause cataclysmic tidal

1 Remarkably, this formula is correct as it stands, even today, and applies to the simplest known black-hole solution of GR, the so-called Schwarzschild black hole.

distortions in their vicinity. Combine these with their typical astrophysical surroundings, such as gaseous clouds, interstellar dust, stars and magnetic fields, and the effect is an environment replete with violent high-energy processes of all sorts, energetic jets, matter accretion and outflow, all accompanied by powerful electromagnetic emission ranging from radio to gamma ray frequencies [11]. These effects transform the surroundings of black holes into some of the brightest sources in the Universe, belying the occult proclivities of the objects that power them. The accretion disk of in-spiraling gas and dust is energised through frictional heating, turbulence and gravitational energy release, and heated to temperatures of millions of kelvins leading to intense UV and X-ray emission (see Fig. 1). Some black holes collimate plasma energy into narrow relativistic jets extending thousands to millions of lightyears and accelerating electrons that emit synchrotron radiation from radio via optical to X-ray frequencies. Combined with the mind-boggling size of supermassive black holes, which can have tens of billions of solar masses, these violent events constitute some of the brightest persistent objects in the known Universe, powering active galactic nuclei including quasars.

Given these physical processes in their vicinity, it becomes possible, in principle, to obtain direct astronomical images of black holes; that is if there exist large-enough black holes that are located close enough to Earth to be above the threshold of observability. Luckily the observational stars align for two sources (so far), namely Sgr A* and M87, two supermassive black holes subtending something like 50 µas in the sky. This makes it possible to obtain direct images in millimeter radio observations, employing very-long baseline interferometry (VLBI), combined with highly sophisticated image processing technology [9, 12], shown in Fig. 2. Current observations make contact with GRMHD simulations (General relativistic magnetohydrodynamics) of accretion and jet formation, comparing to and benchmarking against the polarization signature measured in the emission, visualised in the composites shown in Fig. 2. Future proposed

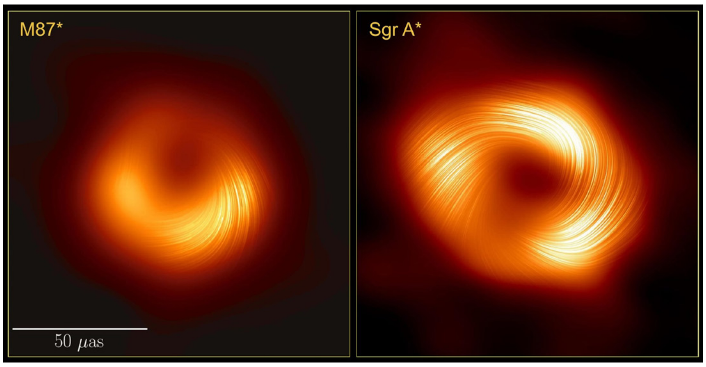

Fig. 2: Shadows of M87 and Sgr A supermassive black holes, [9]. The former is located at the center of galaxy M87, at a distance of 55 million light years, with an event horizon of radius $R_s \approx 130$ AU. Despite its immense size, it subtends only 42 µas in the sky, underlining the sophisticated imaging techniques involved, which have a resolution equivalent to faithfully imaging an orange placed on the Moon. The black shadow visible at the center of the image corresponds to the photon orbit at $R = 2.6\, R_s$, slightly larger than the event horizon at $R = R_s$ itself (see Eq. (1)). The bright emission around the black hole's shadow has its origin in the heated accreting plasma and is in the radio spectrum. The 'wind- like''swirls are a GRMHD assisted extrapolation of measured polarization data (real data at currently realistic resolution can be found in [10].) [Image Credit: EHT Collaboration].*

missions will target more intrinsic GR signatures, such as, for example, the black-hole photon ring [13].

Light from the Abyss: Illuminating the Paradoxes of Black Holes

Typically overwhelmed in magnitude by the astrophysical processes outlined above, there exists another, entirely separate, source of radiation emanating from black holes. This is the celebrated prediction made my Stephen Hawking in 1975, [14], on the basis of a visionary calculation that explored the consequences of quantum effects for the behavior of black holes. Rather than being the ideal absorber that classical GR predicts – the strong emission of its *surrounding environment* notwithstanding – the gravitational effects of a black-hole horizon actually have the effect of conjuring up a faint, perfectly thermal, radiation from the vacuum. This so-called Hawking radiation, however weak, differs dramatically from all other physical processes we have seen in at least two major respects. Firstly, its origin is quantum gravitational [2], and secondly, this effect causes a black hole to shed mass, rather than acquire it, and will thus eventually lead to its evaporation by Hawking radiation. This apparent violation arises because Hawking radiation does not respect the classical energy conditions on which the area theorem rests (see (2) below for an explanation). For a long period of a typical black hole's life the mass loss rate through Hawking radiation would be dwarfed by its mass accretion.

Importantly, this state of affairs leads to a surprising paradox: given that the initial black hole can in principle contain an arbitrary amount of information, say encoded in a specific sequence of (q-)bits of the in-falling matter creating it, the end state of perfect thermal radiation has all but forgotten it: the system is now characterised by only a few numbers, such as temperature and chemical potential, entering the thermal Planck distribution of the radiation left behind. This dynamical "loss of information" violates the fundamental tenet of *unitary* of quantum theory. Ever since Hawking's original work, this effect has been treated as the canary in the coal mine of quantum gravity, signaling the severe tension between the laws of gravity and those of quantum mechanics.

Black holes: from classical thermodynamics to quantum statistics

The temperature, T_H of Hawking radiation constitutes only one aspect of the so-called laws of black-hole thermodynamics, which assign a black hole the entropy, S_{BH}, named after Bekenstein [17], and Hawking [14]. Remarkably, Einstein's equations for stationary horizons imply analogues of the first, second, and zeroth laws of thermodynamics [18]. When these are combined with Hawking's semiclassical result for the temperature, they yield the full set of black-hole thermodynamic relations:

$$S_{BH} = \frac{k_B c^3 A}{4\hbar G_N}, \quad \text{such that} \qquad (2)$$
$$(i)\colon\, d(Mc^2) = T_H\, dS, \quad (ii)\colon\, dS \geq 0,$$

2 The reader worrying how quantum gravity effects could be reliably calculated without having a fully fledged underlying theory may find solace in the knowledge that the calculation can be carried out fully within effective field theory while remaining in low-curvature regions of the black-hole spacetime [15].

together with the zeroth law, stating the constancy of T_H on the horizon of an equilibrium black hole. This temperature is typically very small, clocking in at 0.02 K for a black hole of the mass of the Earth, or an even tinier 60 nK for a solar mass black hole. Remarkably the entropy is proportional to A, the area of the black hole, and not, as would be more usual, its volume; and secondly, that the equations of GR, subject to reasonable assumptions on the kind of matter we expect in our Universe, called energy conditions [3], assure that this area of the event horizon can only increase in any classically allowed process. In other words, this second law is a demonstrable consequence of GR, first shown in [19]. In another instance how a once highly theoretical field has been transformed into experimentally accessible science, this area theorem has been verified in observations of gravitational waves emitted by black-hole mergers, giving results consistent with the assertion that the horizon area of the single black hole produced by binary fusion is greater than the sum of the individual horizon areas pre-fusion [16] (see Fig. 3).

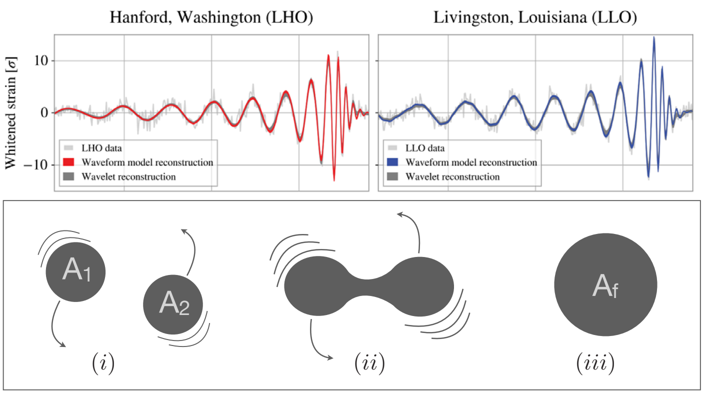

Fig. 3: GW250114 gravitational wave signal observed by the two LIGO detectors (upper panel). The data analysis reveals a black-hole fusion event of two nearly equal mass holes of $m_1 \sim 33.6\ M_\odot$ and $m_2 \sim 32.2\ M_\odot$. By fitting the leading and subleading quasi-resonances (also known as quasinormal modes), [16] confirm Hawking's area law in this merger with high confidence [image credit: LIGO collaboration]. This is illustrated schematically in the lower panel for the case of the merger of two black holes of roughly equal size, starting in a decaying orbit around each other, (i). After the black holes get so close that their horizons touch, (ii), a single new horizon emerges that eventually settles down, (iii), accompanied by the gravitational radiation eventually observed by LIGO. The area theorem implies that the final horizon area is always at least as big as the sum of the initial ones, $A_f \geq A_1 + A_2$. More details on observational and theoretical aspects of GW science can be found in [43] and [44].

Accepting the thermodynamic nature of black holes, one is led to the natural question what microscopic underlying description would produce this behavior as its macroscopic manifestation, as has been so successfully used by Maxwell, Boltzmann, Gibbs and others in order to explain the thermodynamic behavior of gases, or other macroscopic systems. In the present context, this program amounts to identifying the 'microstates' that underlie a black hole's macroscopic behavior, and furthermore to understand the dynamical laws that govern it. In other words, black-hole thermodynamics is at the core of the efforts to formulate and understand quantum spacetime.

3 The least restrictive assumption here being the so-called null energy condition, which states that the energy momentum tensor satisfies $T_{\mu\nu}k^\mu k^\nu \geq 0$ for any null directions k^μ.

In quantum statistical physics we can use the partition sum, $Z = \sum_i e^{-\beta E_i}$ at inverse temperature $\beta = 1/k_B T$, and its associated free energy, $F = -k_B T \ln Z$ to define the entropy, $S = -\frac{\partial F}{\partial T} = k_B(\beta E + \ln Z)$, where E is the average energy in the canonical ensemble. In gravity, as was first shown in [20], one can identify the analog of the partition sum as the Euclidean action of the black hole, that is the Einstein-Hilbert action of the black hole, I_{BH}, whose time variable has been analytically continued $t \to it$, giving

$$Z_{BH} = e^{-I_{BH}/\hbar} \Rightarrow S = k_B\left(\beta E - \frac{I_{BH}}{\hbar}\right). \quad (3)$$

The heuristic justification for the identification of the partition function (see footnote 2 above) with the Euclidean action of the black hole comes from the idea that Euclidean field theory, that is field theory with imaginary time, $t \to it$, is a mathematical reformulation of statistical mechanics [21]. As a consistency check, when one uses the Euclidean action of a Schwarzschild black hole of mass M, which evaluates to $I_{Sch}/\hbar = \frac{1}{2}\beta M c^2$, together with its Hawking temperature and horizon area

$$\begin{aligned} A &= \frac{16\pi G_N^2 M^2}{c^4}, \\ T_H &= \frac{\hbar c^3}{8\pi G_N k_B M}, \end{aligned} \quad (4)$$

the formulae above reproduce the entropy relation stated in (2). Analogous analyses result in more general thermodynamic functions for the other classic families of black-hole solutions, such as Kerr and Kerr-Newman, which consistently enrich the laws (2) with chemical potentials for charge and rotation. One might wonder about a more technical justification for the relation (3). A partial answer is that it follows from a semiclassical approximation of the Euclidean gravitational path integral

$$Z_{BH} = \int \mathcal{D}g_{\mu\nu}\, e^{-\frac{1}{\hbar}S_E[g_{\mu\nu}]}, \quad (5)$$

where $S_E[g_{\mu\nu}]$ is the Wick-rotated Einstein-Hilbert action for the metric tensor $g_{\mu\nu}$ and the integration over $\mathcal{D}g_{\mu\nu}$ indicates a formal Feynman path integral over all possible metric configurations. To the extent this object is well defined [20] (for example in a one-loop expansion about its saddle points), its evaluation in the $\hbar \ll S_E[\bar{g}_{\mu\nu}]$ limit results in the expression (3), where I_{BH} is the value of the Euclidean Einstein action, $S_E[\bar{g}_{\mu\nu}]$, evaluated on its saddle point, $\bar{g}_{\mu\nu}$, that is the classical solution – in the present case, the Wick-rotated Schwarzschild solution. This semiclassical derivation of a true quantum gravitational result, namely the Hawking effect, is remarkable, since nowhere did we have to mention or further characterise the underlying microstates. The results described in this section are thus situated somehwere in between the fully microscopic description à la Maxwell- Boltzmann we sought, and the macroscopic thermodynamics laws of black-hole mechanics. If one wants to go further, and truly interpret S_{BH} one needs to retreat to more theoretically controlled, albeit less realistic, models. In a further landmark result, [22] were able to give a counting of microstates $|\psi_i\rangle$ so that $S_{BH} = k_B(\beta E - \ln Z_{BH})$, where $Z_{BH} = \sum_i e^{-\beta E_i}$ is the partition sum of their microstates. This amounts to a fully microscopic understanding of black-hole thermodynamics, albeit at the expense of having to work in the more exotic theoretical laboratory of five-dimensional supersymmetric black holes, somehwat distant cousins, one might say, of Schwarzschild's 1915 brain child, but nonetheless a landmark proof-of-principle achievement that a microscopic understanding of (2) is possible.

A quantum condition on black holes

Suppose now that the microscopic description of the black hole microstates follows the standard rules of quantum mechanics. In that case we have a global state of the system, $|\Psi\rangle \in \mathcal{H}_{BH} \otimes \mathcal{H}_R$, where *'R'* stands for radiation. In other words we have bi-partitioned the system into degrees of freedom inside the black hole and outside, calling the latter 'radiation'. It is then possible to track the entropy of the radiation, S_R, as a function of time, more specifically its subsystem entropy, also known as the von-Neumann entropy [4] of the reduced density matrix of the radiation $\rho_R = \mathrm{Tr}_{\mathcal{H}_{BH}} |\Psi\rangle\langle\Psi|$. At the beginning of the evaporation process there is no radiation and thus $S_R = -\mathrm{Tr}\rho_R \ln \rho_R$ vanishes. As the black hole radiates, S_R increases at the rate of emission of Hawking quanta. Eventually, by the time all of the black hole has evaporated, S_R must again have decreased down to zero, as the radiation now contains all of the degrees of freedom present in the global state $|\Psi\rangle$, in technical terms, the radiation subsystem is again pure. In between these two extremes there must therefore be a turn-around point where the initially increasing subsystem entropy starts decreasing again (see Fig. 4). A more detailed argument proceeds as follows. Assuming that the global state $|\Psi\rangle$ is *randomly* chosen [5], Page proves, [24], that the subsystem entropy, corresponding to a bi-partition $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$, of the smaller subsystem is near maximal,

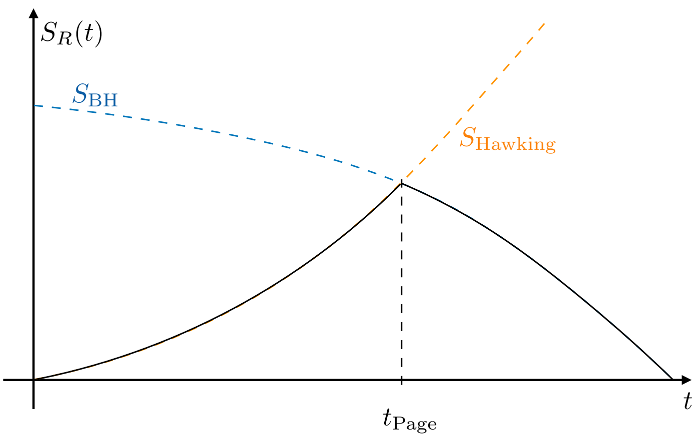

Fig. 4: While the area of the black hole decreases during evaporation, implying a decreasing entropy (see Eq. (2)), Hawking's landmark calculation [14] predicts an ever increasing radiation entropy, $S_{Hawking}$. Page's curve (in black) expresses the fact that unitarity of quantum mechanics forces the true entropy curve to bend back down to zero, once roughly half the black hole has evaporated, which happens at the eponymous Page time. This curve has now been reproduced from semiclassical calculations [23], but a fully microscopic explanation remains an open problem.

4 The von-Neumann entropy is a "fine-grained" entropy, reflecting the amount of quantum entanglement present in a state. This is opposed to a coarse-grained thermodynamical entropy, which averages over the true microscopic correlations.

5 By this we mean that it can be sampled from the Haar measure. This necessitates sufficiently randomising, that is chaotic dynamics of the actual black holes. More on this below.

that is $S_A \approx \ln d_A$ up to exponentially small corrections, for the case that $\dim\mathcal{H}_A = d_A < d_B = \dim\mathcal{H}_B$. Notably this result holds for any unitary quantum system, without reference to its specific microscopic behavior. One can then apply this reasoning to the bi-partition of the evaporating black hole in terms of radiation and the remaining black hole. It follows from the purity of the global state in conjuction with Page's theorem that $S_R = \ln(\min(d_{BH}(t), d_R(t)))$ at all times. The dimensions $d_R(t)$ and $d_{BH}(t)$ are the time dependent dimensions of $\mathcal{H}_{BH}$ and $\mathcal{H}_R$ respectively, during the evaporation process, so that $d_{BH}(t)$ decreases, while $d_R(t)$ increases as evaporation proceeds.

For the subsystem entropy of the radiation one then gets the following picture (see also Fig. 4): at the beginning of the evaporation S_R increases from zero linearly with the number of emitted quanta. Once half the hole has evaporated, so that $S_R \approx \frac{1}{2} S_{BH}$, the black hole takes over as the smaller subsystem, which now bounds the subsystem entropy of the radiation by purity of the global state. Thus S_R decreases down to zero as $S_R = \ln d_{BH}(t)$.

Quantum extremal surface

It was long assumed that the ability to compute the correct behavior of the Page curve would be the signature application of a full microscopic theory of quantum gravity. It was thus met with a degree of surprise when recent developments provided instead a derivation using methods based on the semiclassical path integral approach (5) analogous to the Euclidean derivation of S_{BH} described above. The key insight comes from the fact that in computing the von-Neumann entropy of a subsystem, for example *R* above, there are new saddle solutions, [25–27] that contribute at late time, causing the entropy to dip down, as required by unitarity. The reason why these saddles went un-noticed before comes from their origin in the so-called *n*–fold 'replicated' path integral, which computes $\mathrm{Tr}(\rho_R^n) := Z_n$, from which the microscopic sub-system entropy follows via the limiting procedure $S_R = -\mathrm{Tr}\rho_R \ln\rho_R = -\frac{\partial}{\partial n}\mathrm{Tr}(\rho_R^n)\big|_{n=1}$. The quantity Z_n can be obtained by imposing appropriate boundary conditions on the integration over metrics in (5). The presence of these new connected solutions, the so-called 'replica wormholes', leaves its trace even after the limit $n \to 1$ is taken, resulting in the *generalized entropy formula* for the subsystem entropy of the radiation, [28, 29]

$$S_R / k_B = \frac{c^3 A}{4\hbar G_N} + S_{\Sigma_R \cup \Sigma_I}. \qquad (6)$$

This formula contains a term that resonates with the black-hole entropy formula, Eq. (2), as well as a second new contribution. Unlike in Bekenstein and Hawking's original formula, where the area term is always equal to the area of the black hole, here the area term on which to evaluate the formula is determined via an extremisation process, which also determines the surface $\Sigma_R \cup \Sigma_I$ referred to as the 'island' contribution. It is then found that an exchange of dominance happens, between two possible such surfaces, where early on the second term dominates giving a result equal to the semiclassical radiation entropy, until at the Page time, the first term takes over, and causes the total subsystem entropy S_R to decrease back to zero, as required by unitarity. Thus this gives a semiclassical argument for the Page curve, entirely analogous to the earlier Euclidean argument for the static black-hole entropy. While this is important progress, it does not give any indication about the microscopic mechanism that allows the radiation to unitarise, or in other words the physical process of information recovery at play. Furthermore, the generalised entropy formula and its usage in the computation of the radiation entropy has so far only been fully established in universes that differ asymptotically from our own, in so-called asymptotically anti-de Sitter spaces.

Microphysics of black holes

In complete analogy with the original Euclidean explanation of the static S_{BH}, the semiclassical argument for the quantum Page curve gives strong motivation, but precious little direct evidence, of a microscopic underlying explanation. Ongoing investigations are revealing the fundamental importance of quantum chaotic correlations and their role in (pseudo-)randomising the information contained in black holes [30, 31], as revealed in low-dimensional toy models [32–34], which are being extended towards more realistic scenarios [35–37], albeit still confined to imaginary universes that share some properties of our own, but differ in other respects. The fundamental role played by quantum chaos suggests an intriguing fresh picture on the nature of quantum spacetime underpinning the unreasonable effectiveness of the semiclassical method. In this picture, (semi-)classical gravity is not a simple classical limit of an underlying microscopic theory, but instead an effective ensemble-like description encoding the chaotic spectral correlations present in the microstates. In the context of quantum chaos, this approach, pioneered by Gutzwiller [38, 39], derives statistical descriptions of chaotic quantum systems employing the semiclassical method. Recent work on the central role of quantum chaos in quantum black-hole physics, [32, 33, 40–42], point towards the possibility of a similar justification of semiclassical methods for unitary black-hole evaporation. Much work remains to be done to extend these ideas towards realistic scenarios, but the seeds of an exciting re-evaluation of the quantum nature of black holes are starting to grow into a profound synthesis of gravity, chaos and statistical physics of information.

In this article we have argued that black-hole science is going through a golden age, driven ahead by spectacular experimental advances exploiting entirely new astrophysical messengers and powering ahead thanks to fast-paced advances in observational techniques, computational methods and resources. These advances have established black-hole research as a flourishing experimental science, exploring some of the most daring predictions of classical GR, thought to be out of reach just a few years ago. At the same time, theoretical research on black holes is successfully addressing long-standing mysteries surrounding these fascinating objects, charting its way from mathematically controlled toy models into more and more realistic territory. While current state of the art observational approaches appear unlikely to reveal insights into the quantum realm these works attempt to demystify, one should never discount the potential of experimental surprises that may lie in store. The future of black holes is bright.

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