Potential of Space Based Navigation for Time and Frequency Transfer

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Space Based Navigation for Time and Frequency Transfer

1. Global Navigation Satellite Systems: an overview

2. GNSS observation equation: parameters for time/frequency transfer

3. Time and Frequency Transfer Using Pseudorange Measurements
    - Single satellite synchronisation
    - Multi satellite synchronisation
    - Time/frequency transfer for a baseline

4. Time and Frequency Transfer Using Carrier Phase Measurements
    - Comparison of code and phase measurements
    - Combined code and phase time/frequency transfer
    - Frequency transfer only uing phase measurements

5. Summary

Global Navigation Satellite Systems (GNSS)

- GPS: Global Positioning System
  - operated by the U.S. department of defense
  - Military navigation system, also open for civil users

- GLONASS: Global Navigation Satellite System
  - operated by the Russian military
  - Military navigation system, also open for civil users

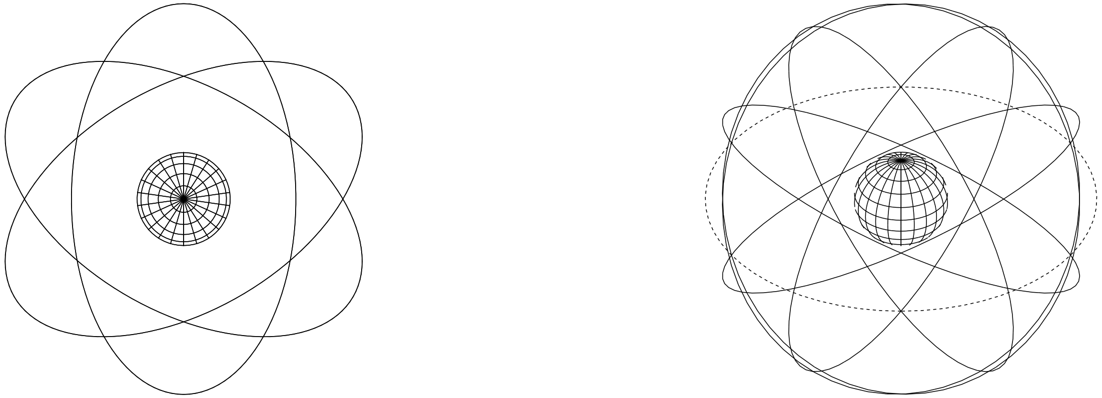
- Galileo
  - will be operated by a commercial company
  - The first civil navigation system.

- other systems
  - there are some plans for national navigation systems
  - example: Japan, India, China

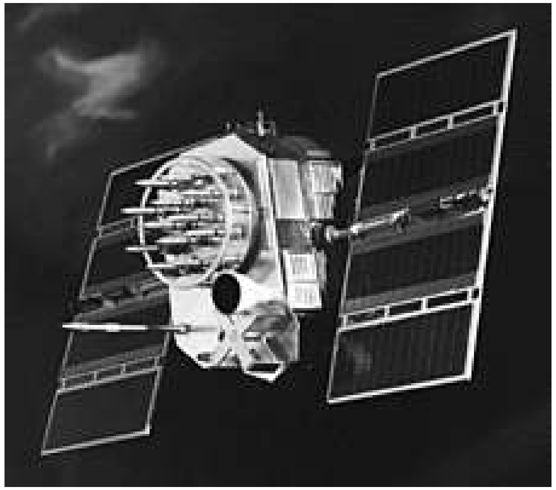
Global Positioning System (GPS)

- The orbits are almost circular, the semimajor axes are $a \approx$ 26’500 km, the inclinations are $i \approx$ 55°. The orbital planes are separated by about 60° on the equator.
- The full constellation theoretically consists of 21 satellites and 3 active spares.
- At present 31 satellites are active including three modernized GPS satellite.

  (PRN: 12, 17, 31)
- The Orbital Planes of the GPS from the poles and from a latitude of 35°:



Global Positioning System (GPS)



Block-II Satellite.

- GPS Satellites are big structures:

  The solar panels have a size of ca. 3.2 m $\times$ 1.7 m, the mass is about $m \approx$ 1 000 kg.

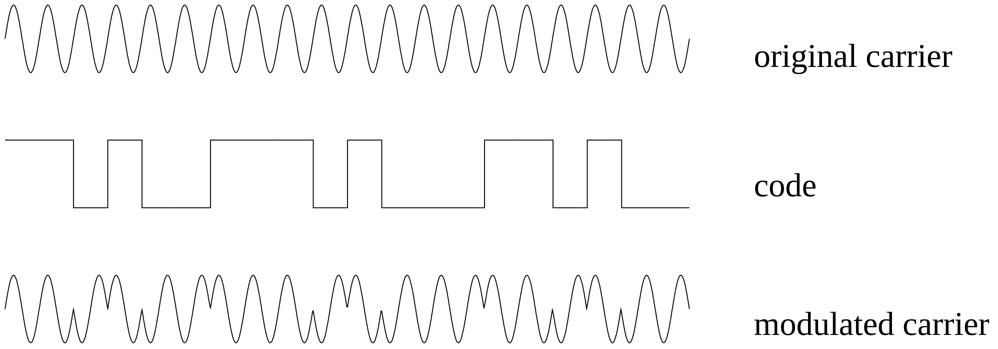
- Orbit modeling is not easy due to radiation pressure.

- Each satellite has a (series of) oscillator(s) generating two coherent carrier phases in the L-band with frequencies of

  L1: f_1 =1.57542GHz, L2: f_2 =1.2276GHz

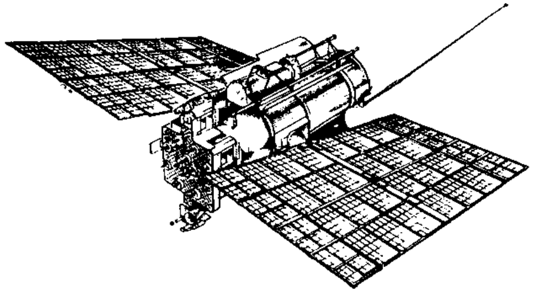
Global Positioning System (GPS)

- Information is sent by phase modulation:



- Broadcast Ephemerides, Satellite Clock Corrections (to GPS system time), C/A—Code, and P— or Y—Code are emitted by the satellite. The first three items are only transmitted on L1.
- C/A code (Clear Access Code, generally available) allows to compute pseudorange with about 3 m accuracy, P— and Y—codes with about 0.3 m accuracy.
- P—code is the Precise or Protected code, Y—code is the encrypted version of the P—code.
- L1 and L2 are right-handed circularly polarized.
- $phase \times c$ may be reconstructed with an accuracy of $\sim$1 mm!

Global Navigation Satellite System (GLONASS)



GLONASS Satellite.

- Nominal Number of Satellites: 24

  (at present: 19 satelites, including 3 in commissioning phase, 3 unusuable, and 3 inactive)
- Number of Orbital Planes: 3
- Inclination: 64.8°, semimajor axes: 25’510 km
- Carrier frequency L1: $f_1^n = (1602 + n \cdot 0.5625)$ MHz

  Carrier frequency L2: $f_2^n = (1246 + n \cdot 0.4375)$ MHz
- Broadcast ephemerides: Position, velocity, acceleration in Earth-fixed System PZ$-$90 every 30 min.
- Glonass System time: UTC (SU), contains leap seconds!
- Last tripple launch successful on December 25, 2006

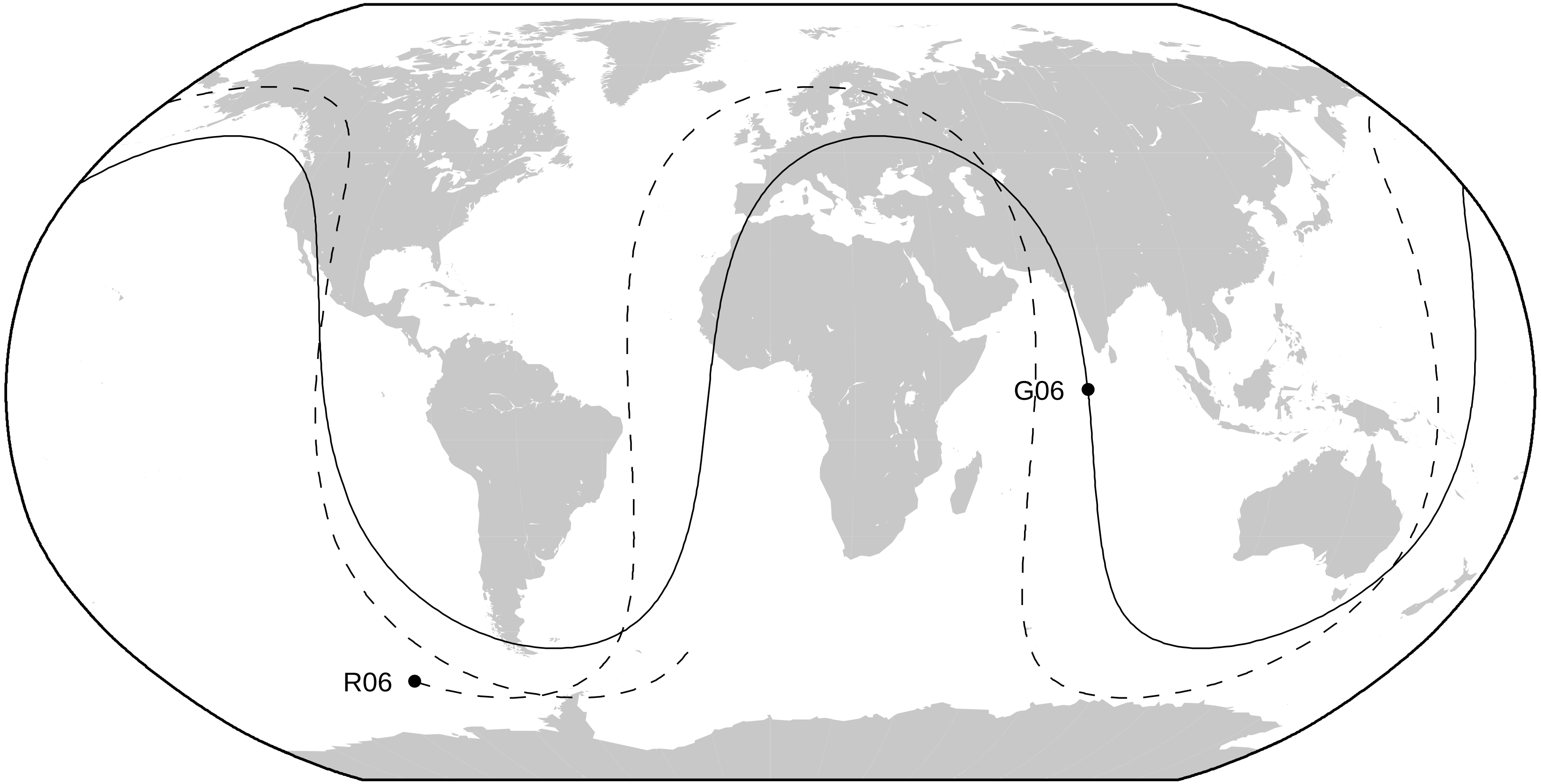
Comparison (GPS/GLONASS)

	GPS	**GLONASS**
Nominal number of satellites	24	24
Operational satellites (February 2007)	31	19 (16)
Orbital planes	6 (separated by 60°)	3 (separated by 120°)
Satellites per orbital plane	4 (unequally spaced)	8 (equally spaced)
Orbital radius	26’560 km	25’510 km
Inclination of orbital planes	55°	64.8°
Revolution period	$\sim$ 11 h 58 min	$\sim$ 11 h 16 min
Ground track repeatability	after one sidereal day	after eight sidereal days
Constellation repeatability	$\sim$ 23 h 56 min	$\sim$ 23 h 56 min
Signal separation technique	CDMA	FDMA
Carrier L1 (n=1...12)	1575.42 MHz	1602.5625 – 1608.75 MHz
Carrier L2 (n=1...12)	1227.60 MHz	1246.4375 – 1251.25 MHz
C/A-code (L1)	1.023 MHz	0.511 MHz
P-code (L1,L2)	10.23 MHz	5.110 MHz
Reference system	WGS—84	PZ—90
Time reference	UTC (USNO)	UTC (SU)

Comparison (GPS/GLONASS)

Ground track of a GPS (G06) and a GLONASS (R06) satellite

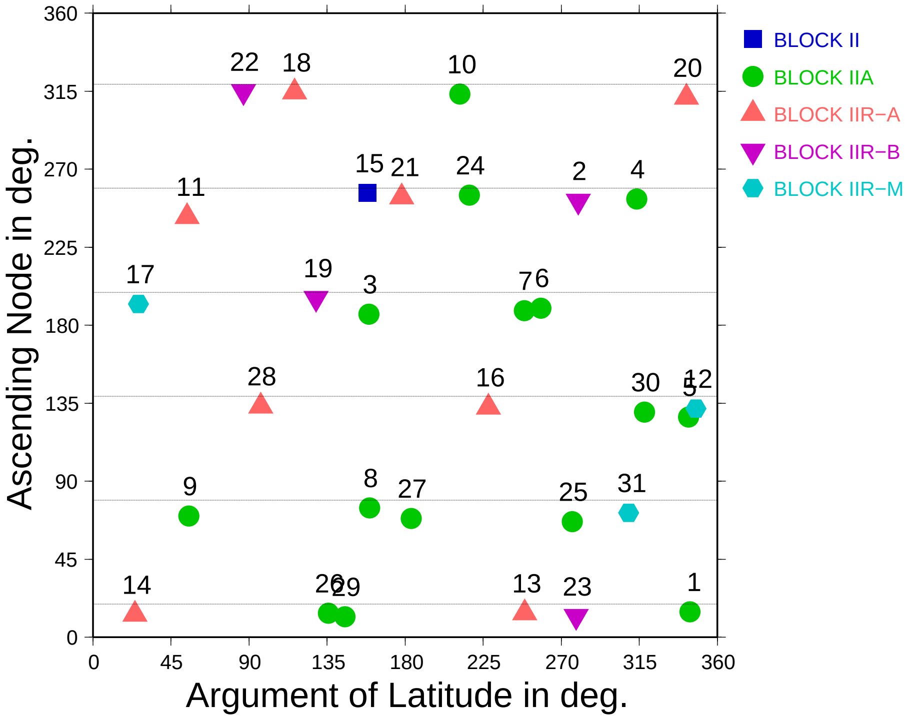
July 7, 2006

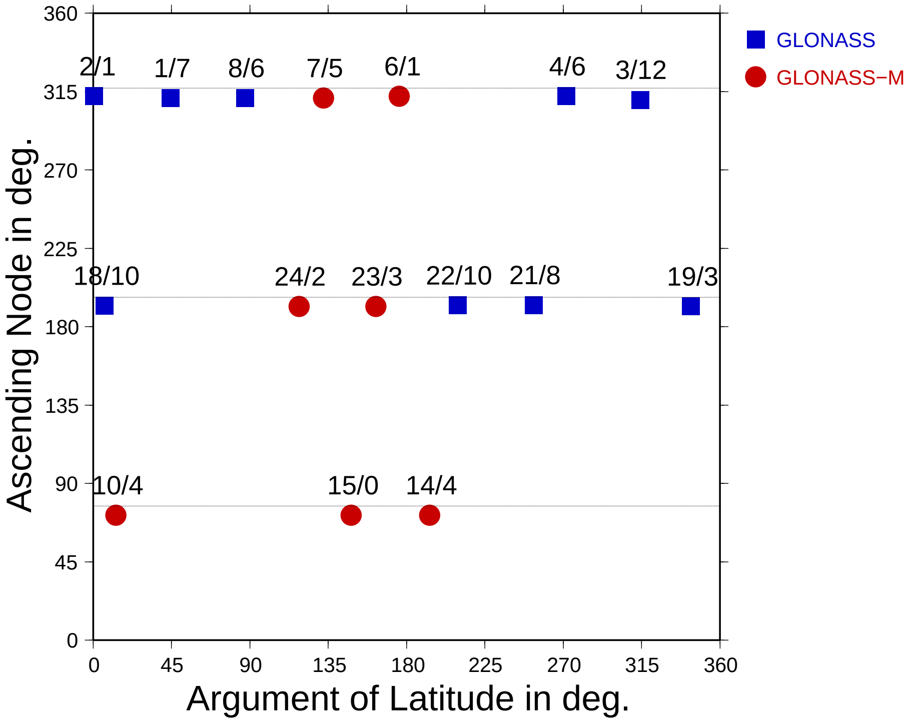


Comparison (GPS/GLONASS)

Constellation Status

February 23, 2007





GNSS Observation Equation

The basic observable is the pseudorange P_i^k, the difference of the reception time t_i of a particular signal (measured in the time frame of the receiver) and the emission time τ_i^k of the same signal at satellite k (measured in the time frame of the satellite):

$$P_i^j = c \cdot (t_i - \tau^k)$$

Assuming an Earth without atmosphere, receivers and satellites with perfectly synchronized clocks, the pseudo-range is equal to the slant range between satellite (at time τ^k) and receiver (at time t_i).

$$P_i^j = \left| \vec{x^k}(\tau^k) - \vec{x_i}(t_i) \right|$$

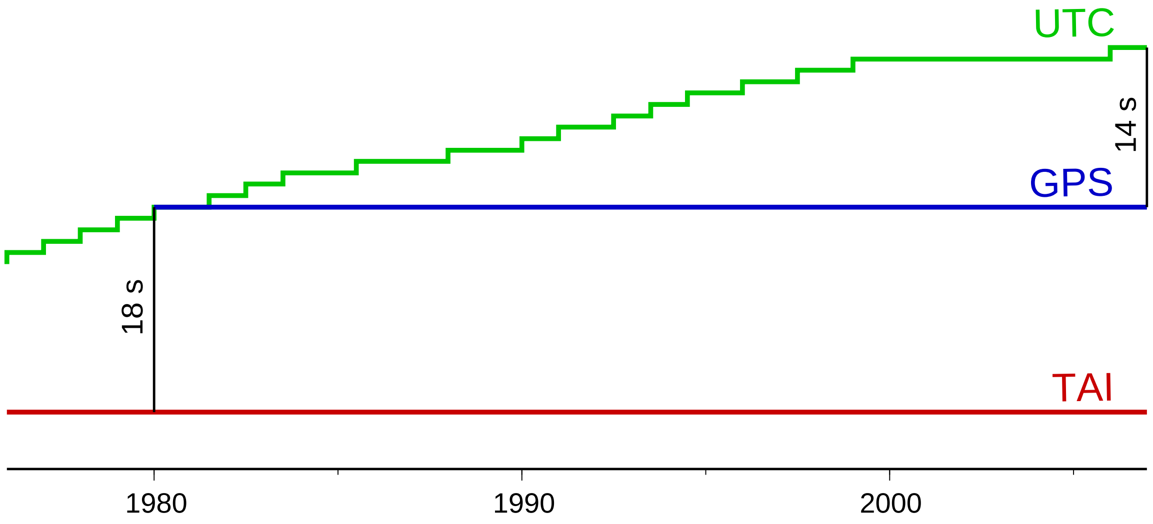
GNSS Observation Equation

$$P_i^k \quad = \quad \left| \vec{x^k} - \vec{x_i} \right| + \Delta_{trop_i}^{\;k} + \Delta_{ion_i}^{\;k} + c\delta_i - c\delta^k$$

P_i^k		Code observation of station i to satellite k
$\vec{x}^{\,k}$	broadcasted	Position vector of satellite k
$\vec{x}_i$	unknown	Position vector of station i
$\Delta_{trop_i}^{\;k}$	≈ 2.30 m	Signal delay in the troposphere
$\Delta_{ion_i}^{\;k}$	broadcasted	Signal delay in the ionosphere
δ^k	broadcasted	Clock correction of the transmitter of satellite k with respect to GPS time
δ_i	unknown	Clock correction of the receiver at the station i with respect to GPS time
c	defined	Speed of light

Single Satellite Synchronization

- synchronization to GPS system time and not to UTC:



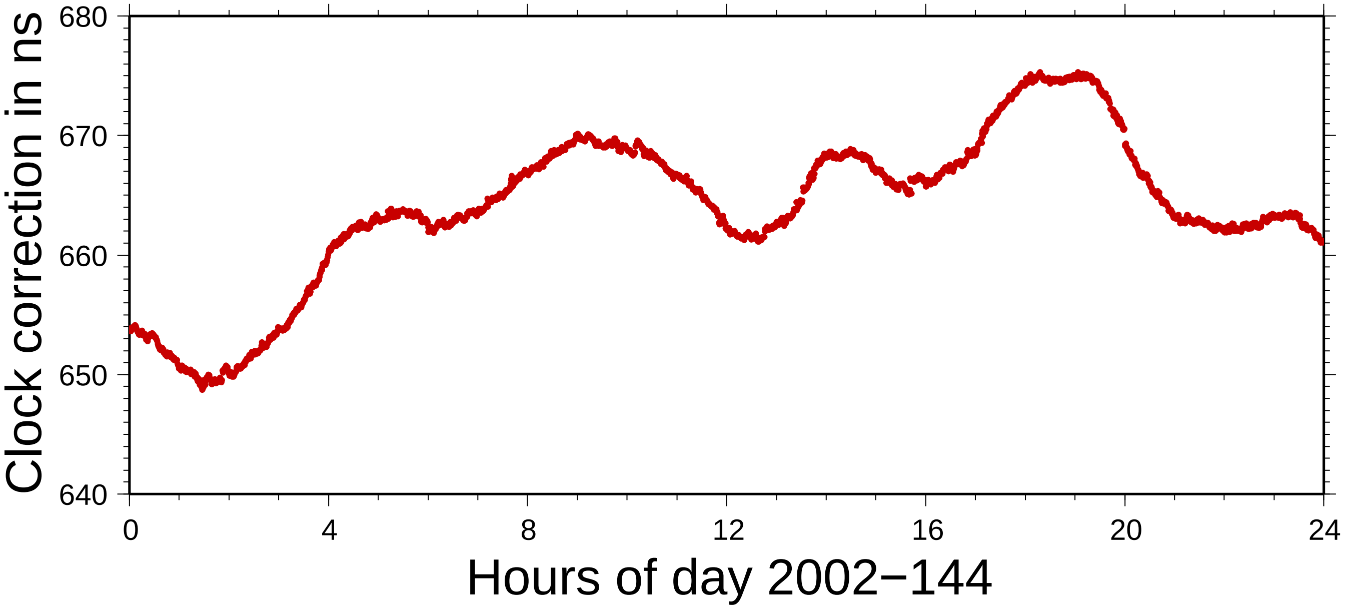
- Advantage: Only one observation at the time
  - single channel receiver, no inter-channel biases
  - simplest possible GPS receiver

- Disadvantage: Only one observation at the time
  - more observations promise better results ($\sqrt{n}$–law)
  - limited by the uncertainty of the broadcasted GPS system time

Multi Satellite Synchronization

- Multiple satellites are used for the synchronization.
  - use of a multi–channel receiver that is able to track all satellites in view
    inter–channel calibration within the receiver must be solved

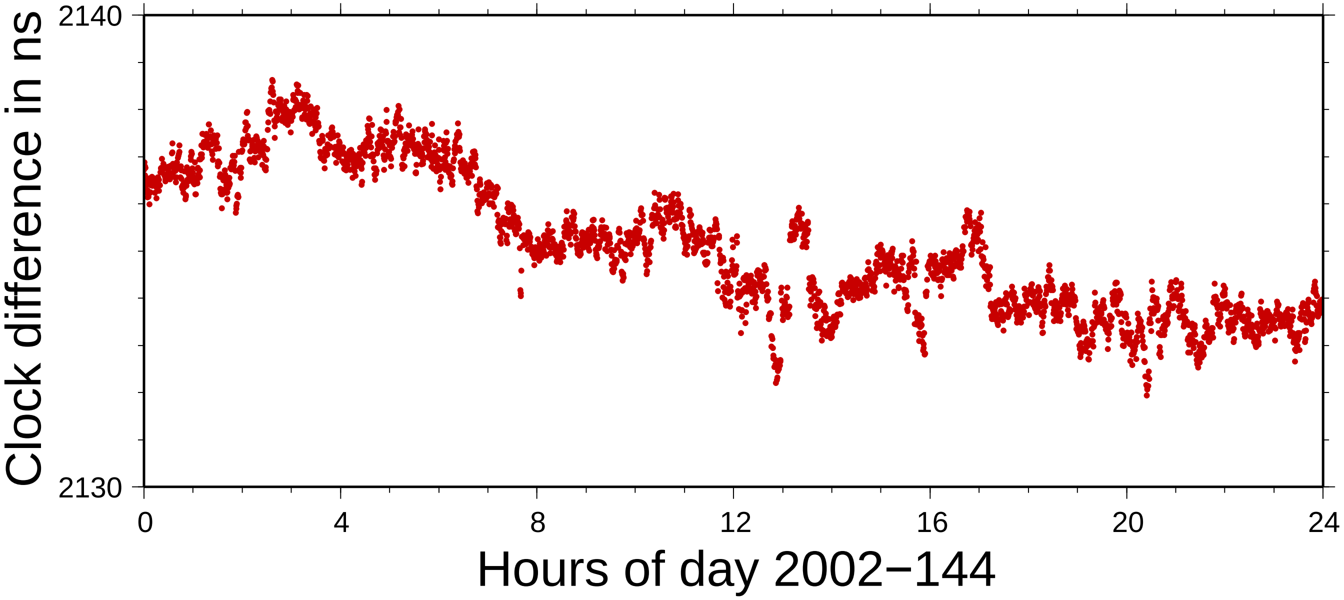
- Example for a multi satellite synchronization for Brussels:



Baseline-wise clock comparison

- Clock differences between two stations (baseline):
  - common error sources cancel out

- Example for a multi satellite clock comparison for the baseline Braunschweig — Brussels:



Baseline-wise clock comparison

- Clock differences between two stations (baseline):
  - common error sources cancel out

- Differences between two stations in the observation equation:

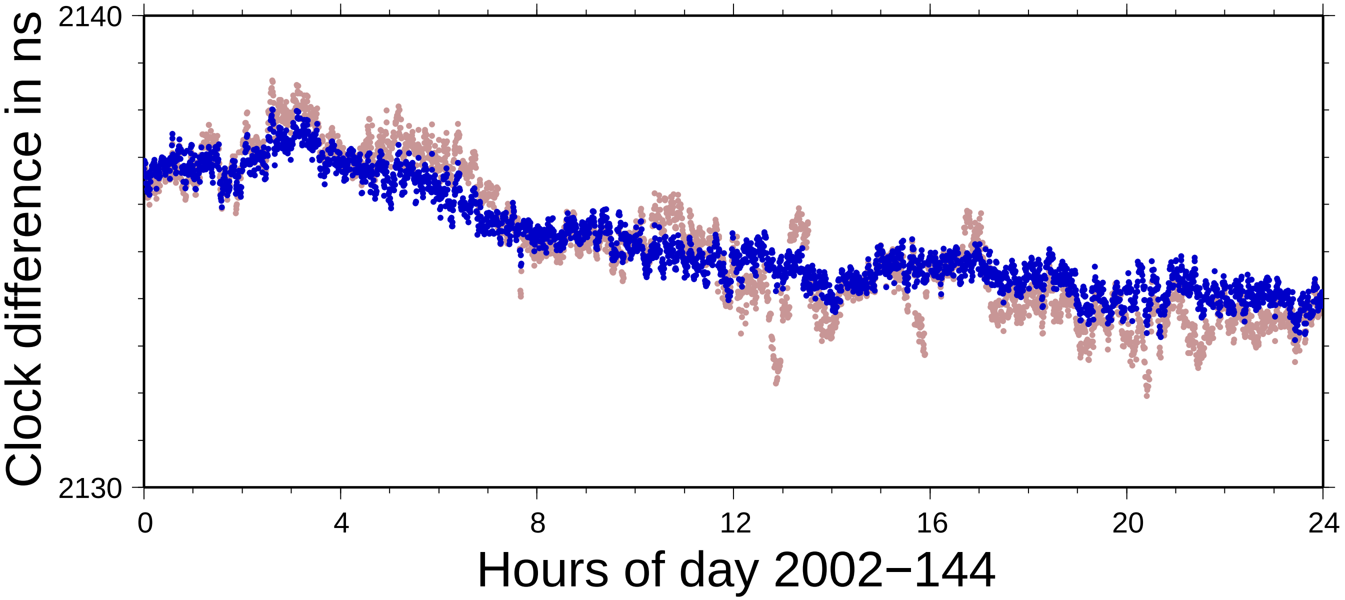
$$
\begin{aligned}
P_i^k &= \left|\vec{x^k} - \vec{x_i}\right| + {\Delta_{trop}}_i^k + {\Delta_{ion}}_i^k + c\delta_i - c\delta^k \\
P_j^k &= \left|\vec{x^k} - \vec{x_j}\right| + {\Delta_{trop}}_j^k + {\Delta_{ion}}_j^k + c\delta_j - c\delta^k \\
P_{ij}^k &= \left|\vec{x^k} - \vec{x_j}\right| - \left|\vec{x^k} - \vec{x_i}\right| + {\Delta_{trop}}_{ij}^k + {\Delta_{ion}}_{ij}^k + c \cdot (\delta_j - \delta_i)
\end{aligned}
$$

- Only satellites that are commonly observed by both stations have to be considered for the processing.
  - for single-satellite observations this require a special organzation effort.

Baseline-wise clock comparison

- Clock differences between two stations (baseline):
  - common error sources cancel out

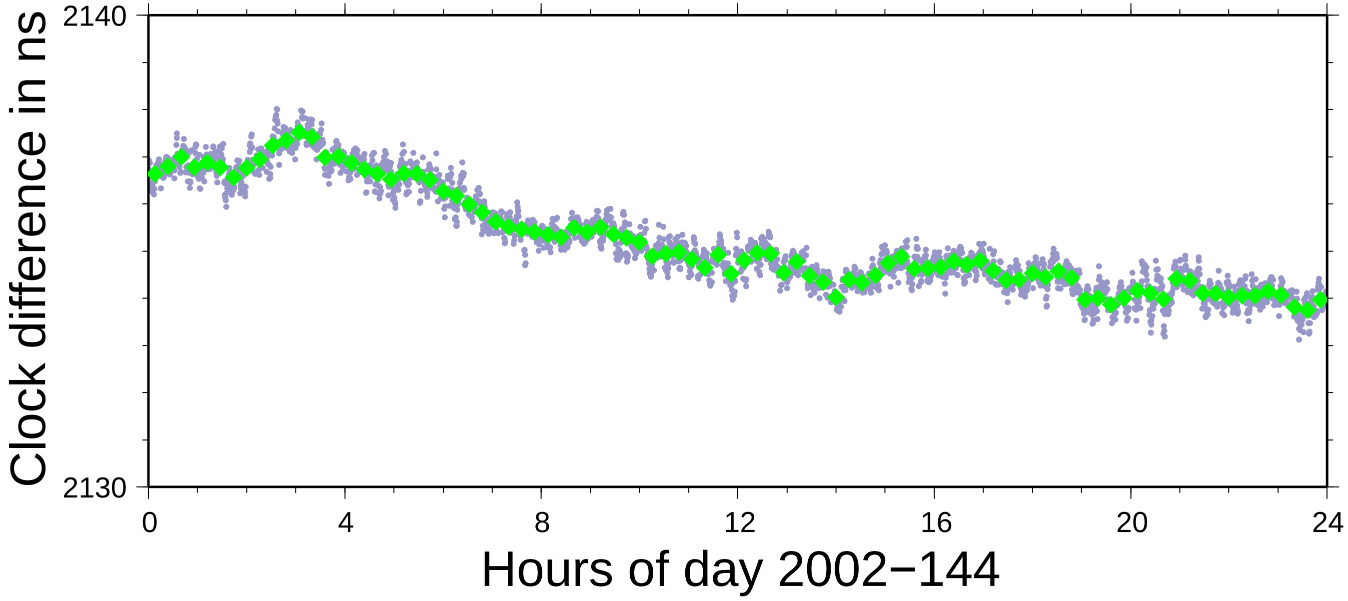
- Example for a multi satellite clock comparison for the baseline Braunschweig—Brussels:



Baseline-wise clock comparison

- The clock differences between two stations may be smoothed for a certain interval:
  - A perfect behaviour of the clocks to be compared is assumed.
  - For an interval length of 16 minutes one obtains 90 values per day.

- Example for a multi satellite clock comparison for the baseline Braunschweig—Brussels:



GPS code based time transfer for TAI

- Smoothed C/A–code time transfer is widely used within the timing community.

- Percentage of time transfer links contributing to TAI
  - 28% C/A–code single channel receivers (Common view to only one satellite)
  - 39% C/A–code multi channel receivers (Common view to multiple satellites)

- Time transfer results are affected by:
  - uncertainty of the broadcast satellite orbits
  - influence of the ionosphere (use dual frequency data)

Dual-frequency clock comparison

- The ionosphere is a dispersive medium for microwaves. Ionospheric refraction may be approximated for carrier Li as:

$$\Delta_{ion\,f} = \frac{\alpha}{f^2} \cdot E$$

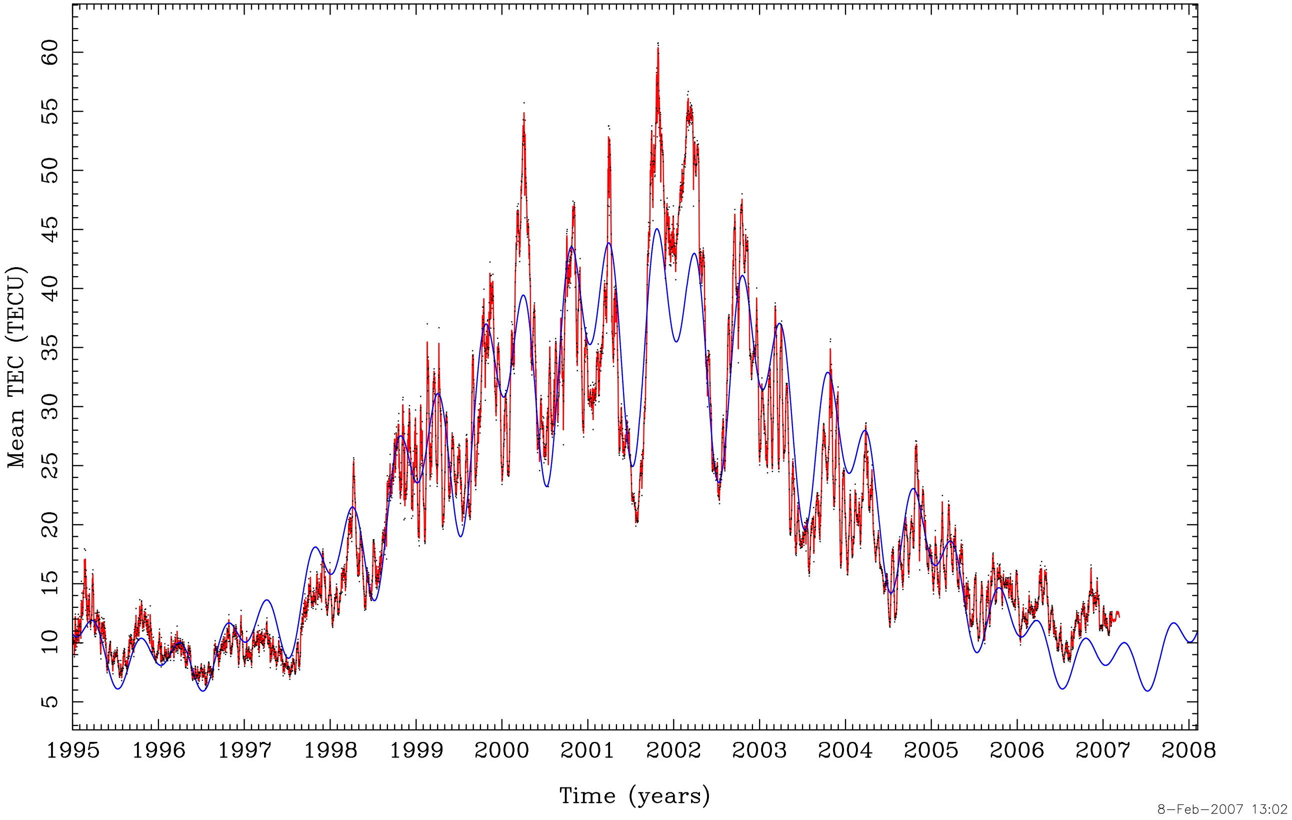
- Observation equations for the two frequencies:

$$\begin{aligned}
P_1{}_i^k &= \left|\vec{x^k} - \vec{x_i}\right| + \Delta_{trop}{}_i^k + \Delta_{ion\,1}{}_i^k + c\delta_i - c\delta^k \\
P_2{}_i^k &= \left|\vec{x^k} - \vec{x_i}\right| + \Delta_{trop}{}_i^k + \Delta_{ion\,2}{}_i^k + c\delta_j - c\delta^k \\
P_2{}_i^k &= \left|\vec{x^k} - \vec{x_i}\right| + \Delta_{trop}{}_i^k + \frac{f_2^2}{f_1^2} \cdot \Delta_{ion\,1}{}_i^k + c\delta_j - c\delta^k
\end{aligned}$$

- Forming a ionosphere–free linear combination:

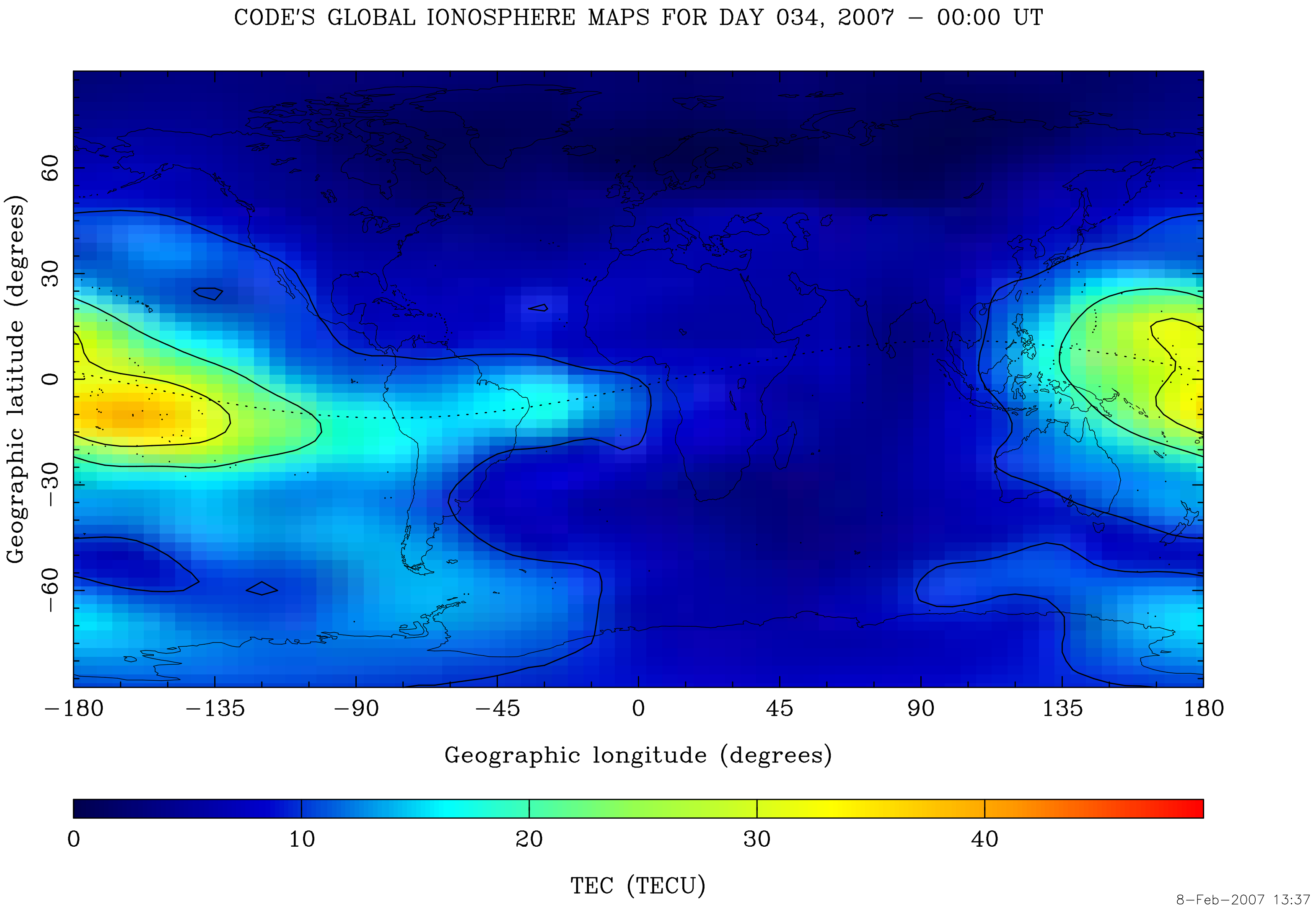
$$\begin{aligned}
P_3{}_i^k &= \frac{f_1^2}{f_1^2 - f_2^2} \cdot P_1{}_i^k - \frac{f_2^2}{f_1^2 - f_2^2} \cdot P_2{}_i^k \\
P_3{}_i^k &= 2.5457 \cdot P_1{}_i^k - 1.5457 \cdot P_2{}_i^k
\end{aligned}$$

GNSS and Ionosphere



Mean TEC since January 1, 1995 estimated by CODE AC, covering a full Solar activity cycle.

GNSS and Ionosphere



GPS Types of Measurements

Measurement types:

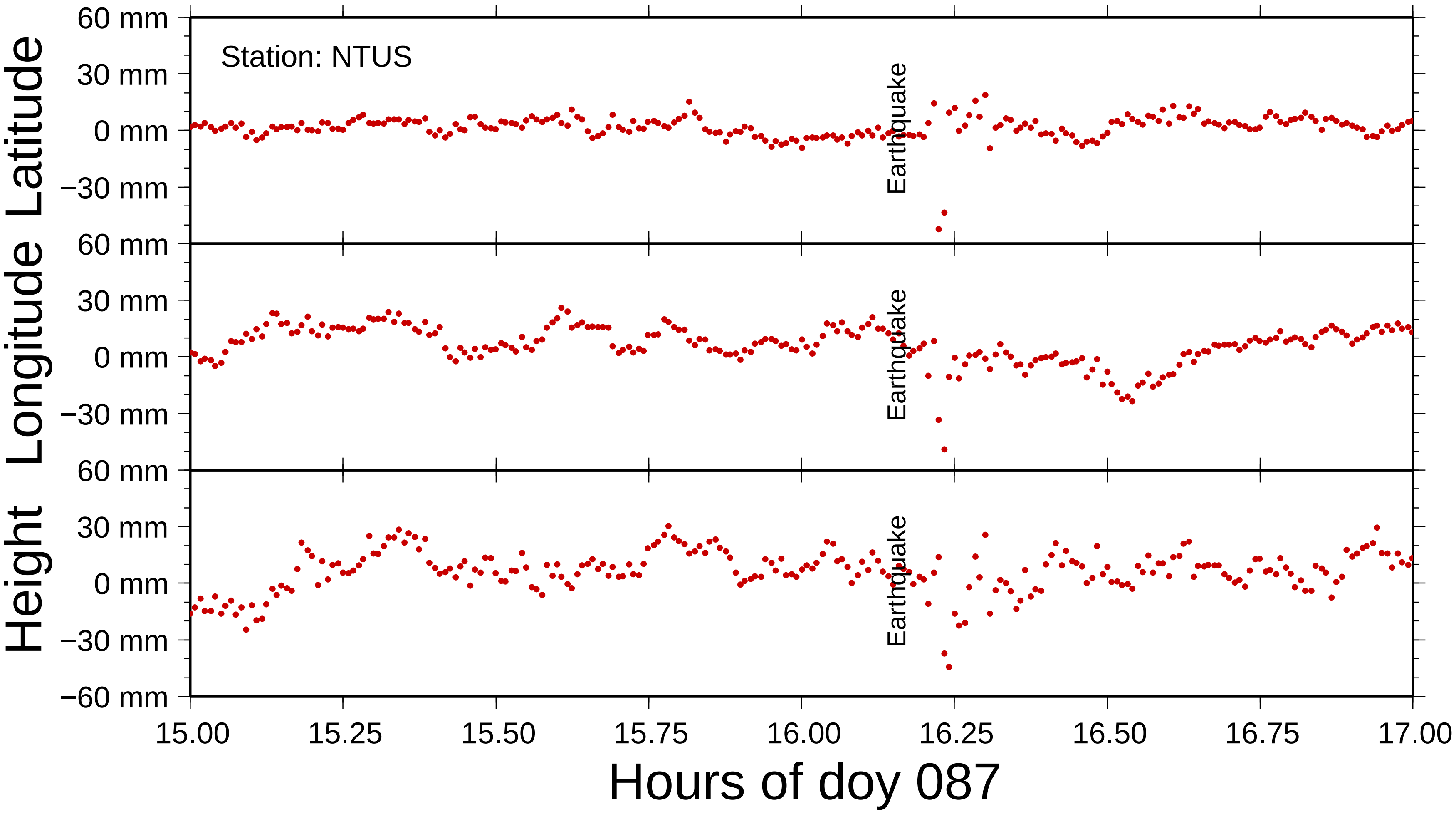
- **C/A Code** on L1 (noise 10 – 100 m or 33 – 333 ns)
- **P-Code** on L1 and L2 (noise 0.2 – 10 m or 0.6 – 33 ns)
- **phase** on L1 and L2 (noise 0.001 – 0.01 m or 0.003 – 0.03 ns).

Phase measurements:

- measurement of cycles (entire cycles and fractionals) of the carrier phase
- resolution of better than 1% of the wavelength
- problem: unknown (integer) number of cycles between the satellite and the receiver at the beginning of the measurement (inital phase ambiguity)
- possible cycle slips: error in counting the entire cycles during a mesaurement (need for detection and correction)
- provided by high–end geodetic type receivers.

Motivation to Use Carrier Phase Data

Kinematic Solution during an Earthquake



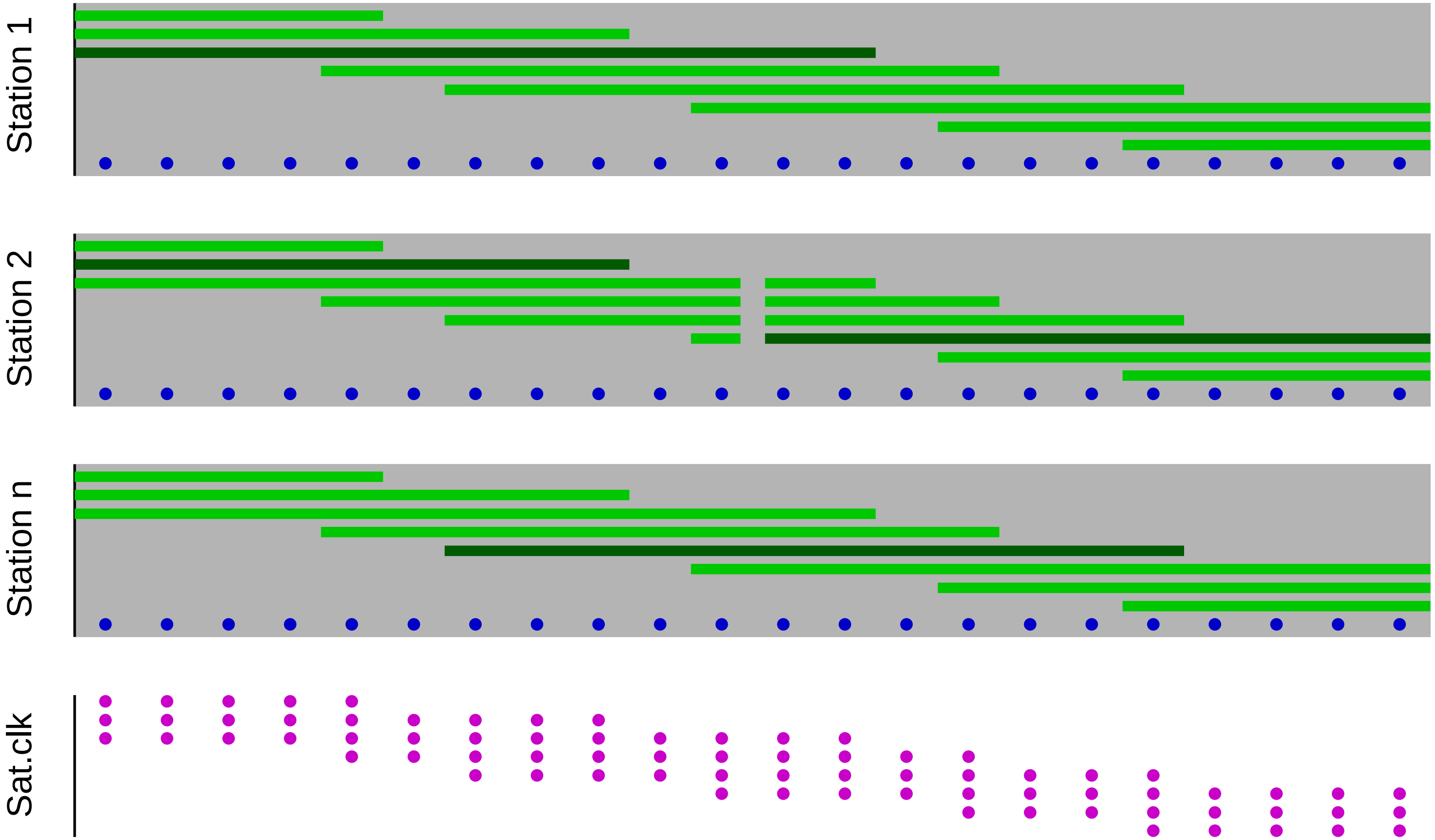
GNSS Observation Equations

$$\begin{aligned} P_i^k &= \left|\vec{x}^k - \vec{x}_i\right| + {\Delta_{trop}}_i^k + {\Delta_{ion}}_i^k + c\delta_i - c\delta^k \\ L_i^k &= \left|\vec{x}^k - \vec{x}_i\right| + {\Delta_{trop}}_i^k - {\Delta_{ion}}_i^k + c\delta_i - c\delta^k + \lambda N_i^k \end{aligned}$$

P_i^k, L_i^k	Code/phase observation of station i to satellite k
$\vec{x}_i, \vec{x}^k$	Position vector of station i and satellite k, respectively
${\Delta_{trop}}_i^k$	Signal delay in the troposphere
${\Delta_{ion}}_i^k$	Signal delay in the ionosphere
δ_i, δ^k	Clock correction of the receiver at the station i, and transmitter of satellite k with respect to GPS time
c	Speed of light
N_i^k	Phase ambiguity (one and the same for one pass)
λ	Wavelength of the carrier phase

Geodetic Time and Frequency Transfer

Principle of the Geodetic Time and Frequency Transfer



Geodetic Time and Frequency Transfer

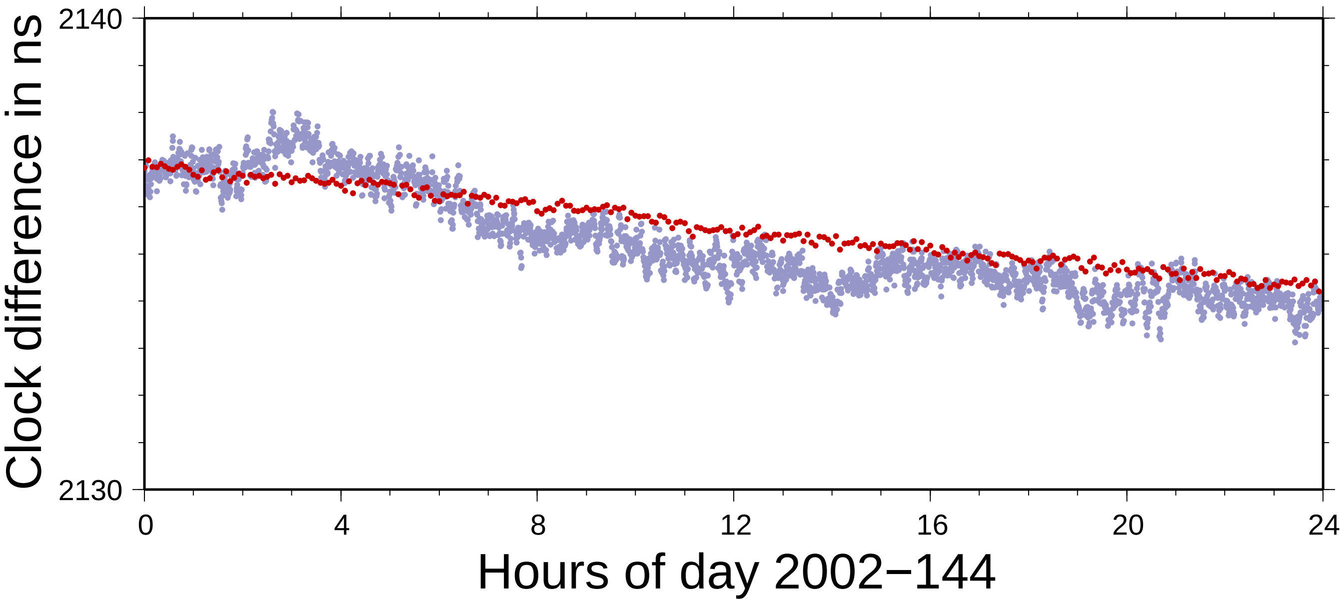
Characteristics of the Geodetic Time and Frequency Transfer

- The complete set of receiver clock values refers to one and the same epoch.
- Only differences between estimated receiver clocks can be interpreted.
- Any simultanious receiver clock difference (baseline) can be extracted from the network solution.
- The method implies no presumptions for the receiver clocks.
- From carrier phase we can get only the change of the receiver clock in time.
- An interrupt in the ambiguities for all satellites leads to a discontinuity in the carrier phase solution.

Geodetic Time and Frequency Transfer

- The pseudorange observations may be added because they have a direct access to the receiver clock parameters.
  - Even in that case the loose of the ambiguity information at one epoch reflects in a discontinuity in the resulting time series or at least in the uncertainty for an obtained frequency.

- Example for a geodetic-style clock comparison for the baseline Braunschweig—Brussels:

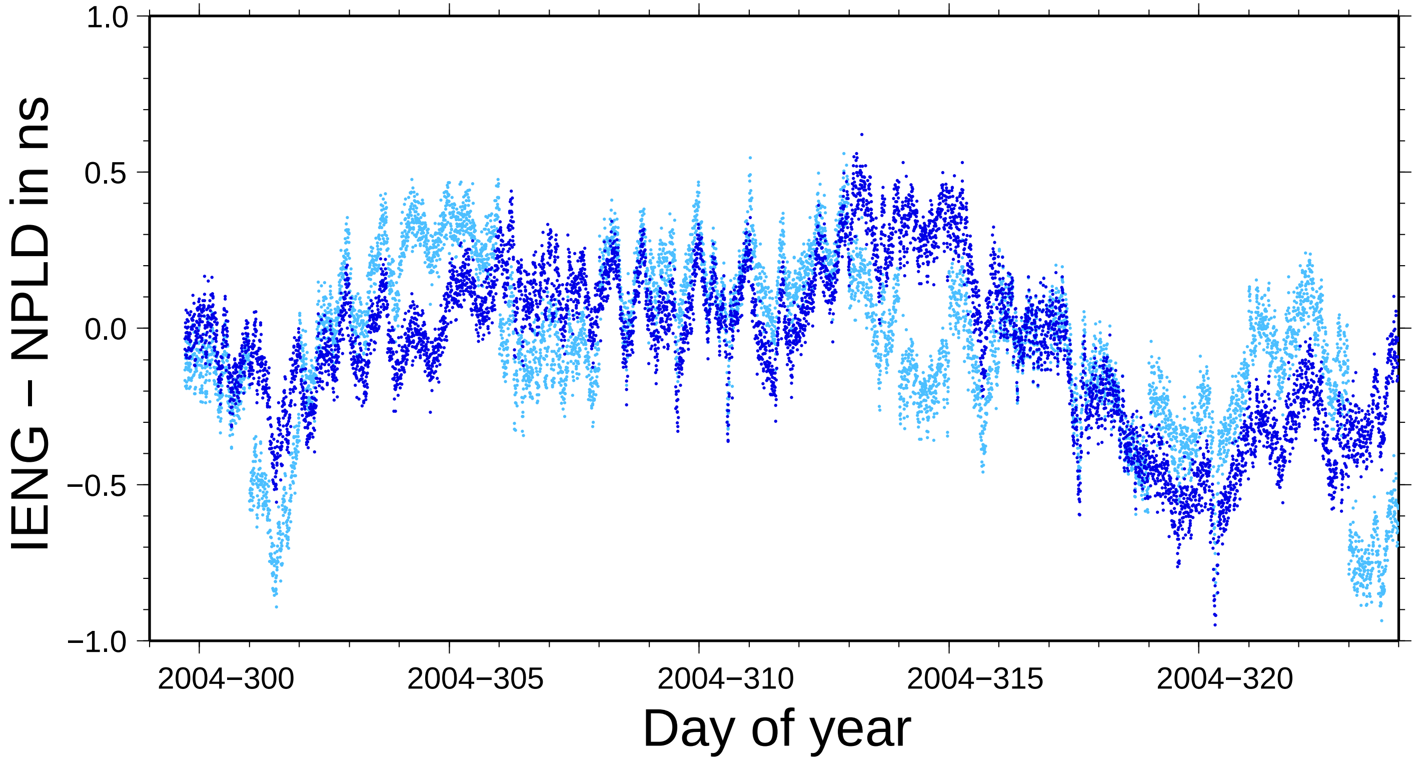


Geodetic Time and Frequency Transfer

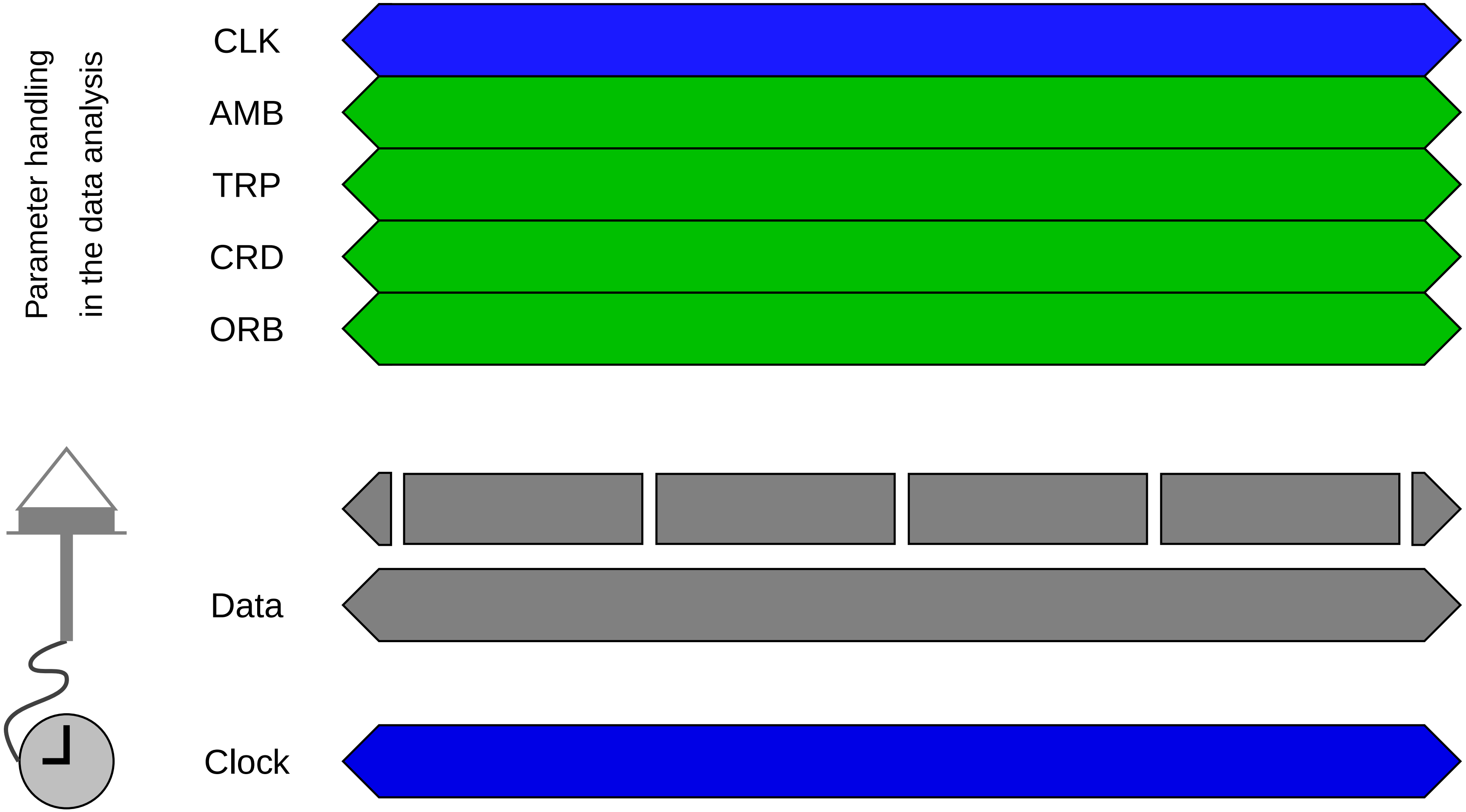
Example for a geodetic time transfer for the baseline Teddington—Torino:

Daily independent solution using code and phase data

Continuous solution using code and phase data



Ambiguity Stacking



Parameter handling
in the data analysis

CLK

AMB

TRP

CRD

ORB

Data

Clock

Ambiguity Stacking

Ambiguity stacking – what are the problems?

- Huge number of parameters
  - nobody is really interested in the ambiguity estimates
  - usually preeliminated as soon as possible

- Mathematically simple but involved bookkeeping in programs
  - keywords: station, satellite, freq./lin. comb., wavelength factor, . . .
  - cycle slips between NEQs
  - initialization of the ambiguities considering phase-windup

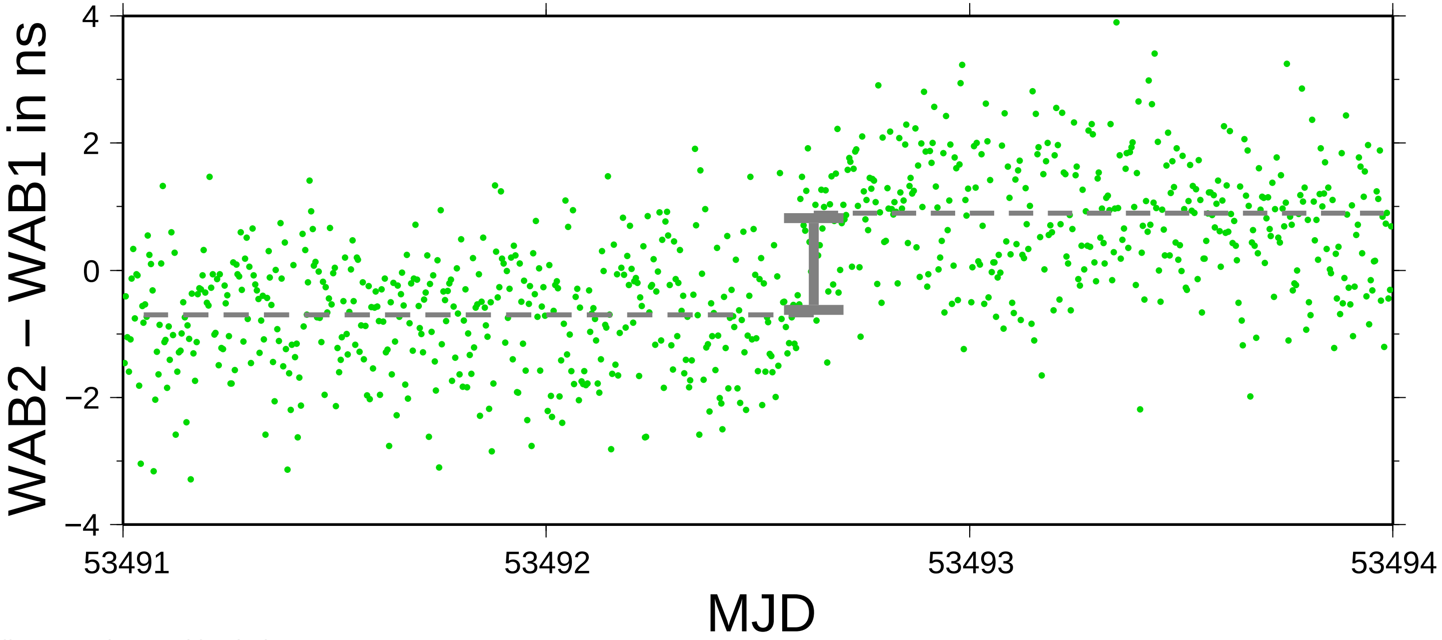
- Advantages
  - may be stacked even if some observations at the boundaries are absent
  - no relative constraints between parameters are necessary

Example: Local Baseline in Wabern

Differences for a local baseline in Wabern

epoch–wise solution only using code data

and continuous solution only using phase data



An offset was substracted for plotting.

Geodetic Frequency–Transfer Solution

Continuity in the phase data can be recovered with the ambiguity stacking as far as it is not disturbed, e.g., by the loss of lock to all satellites.

If no access to the absolute clock value is required (frequency transfer)
the use of the code data is *not* necessary anymore!

Benefit from a phase–only solution is obvious:

- Geodetic receivers are primarily designed for using the phase observations.
- Inconsistencies between the internal receiver clock for code and phase measurements have no effect on the solution.
- Multipath and related effects in the phase data is much smaller than in the code data.

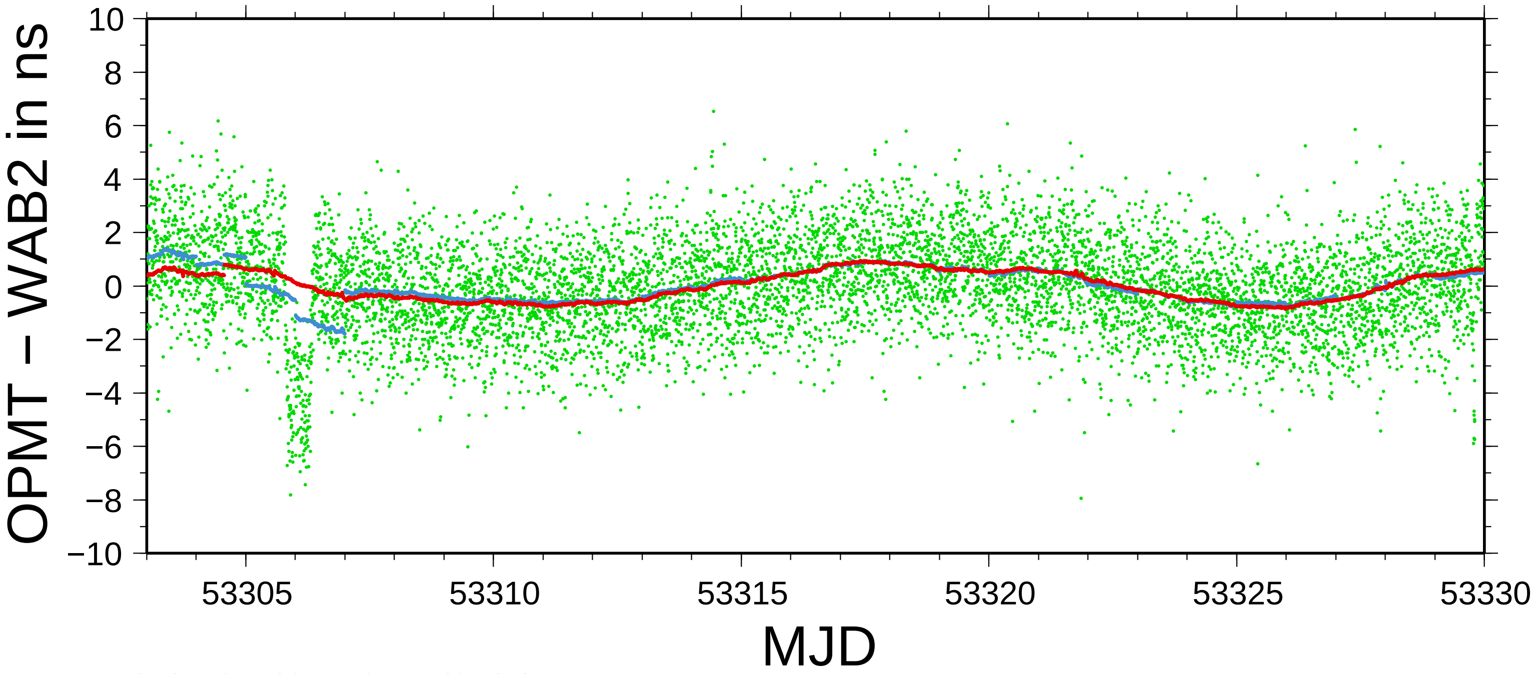
Comparison of Different Solutions

Receiver clock difference between Paris and Wabern

Epoch–wise solution only using code data

Daily independent solution using code and phase data

Continuous solution only using phase data



A common second order polynomial was substracted for plotting.

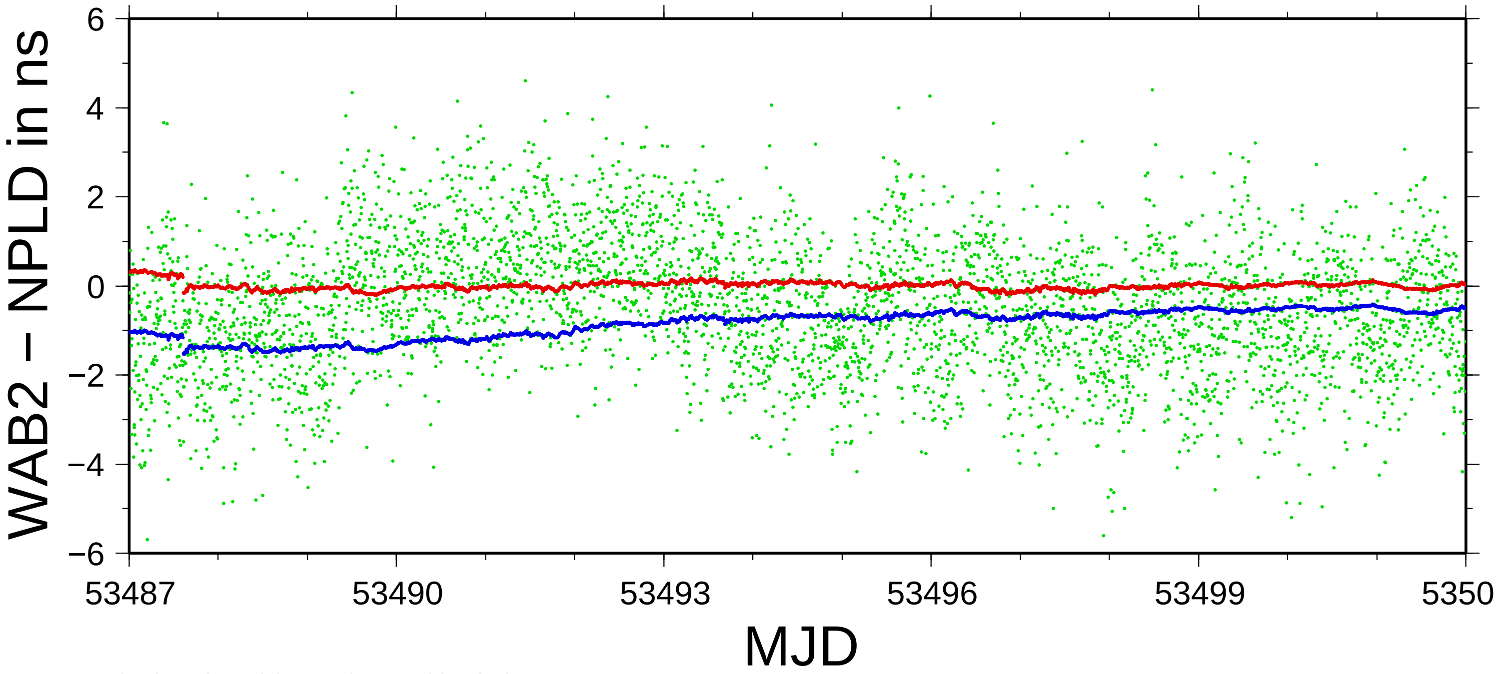
Comparison of Different Solutions

Receiver clock difference between Teddington and Wabern

Continuous solution only using phase data

Epoch–wise solution only using code data

Continuous solution using code and phase data



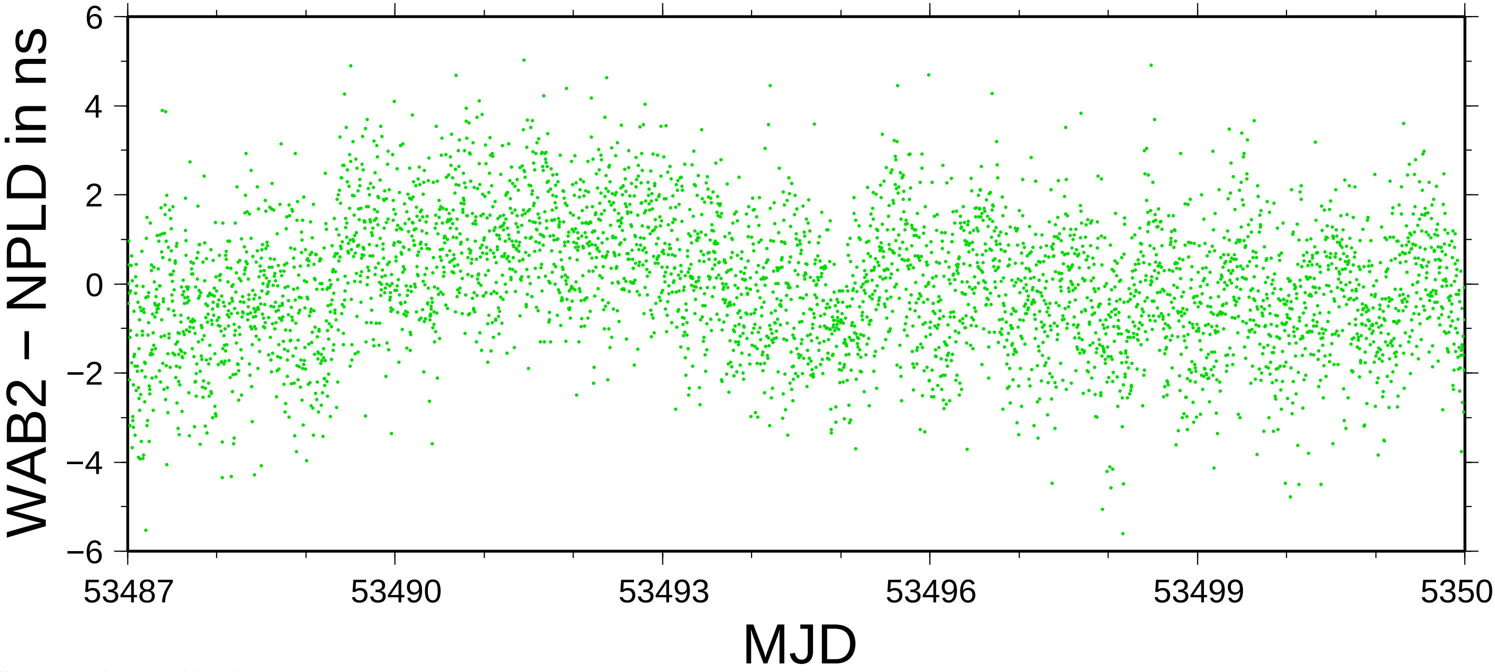
A common second order polynomial was substracted for plotting.

Impact of Multipath

Differences for the baseline Teddington and Wabern

epoch–wise solution only using code data

and continuous solution only using phase data



An offset was substracted for plotting.

Geodetic Time and Frequency Transfer

- To benefit from the high accuracy of the carrier phase measurements an adequate modeling is required:
  - orbit modeling
  - troposphere modeling
  - signal propagation
  - station displacement

- The *International GNSS Service* supports the analysis of GNSS data with high accuracy requirements with its network of stations (observation data) and products:
  - GNSS satellite orbits and clocks,
  - troposphere and ionosphere modells,
  - station coordinates, and
  - receiver clock corrections (IGS time scale).

  Three product lines with different latencies are provided: final (2 weeks), rapid (18 hours), ultra–rapid (3 hours). The ultra–rapid orbits contain a predicted part for real–time applications.

Summary on GNSS Supported T/F Transfer Methods

- Synchronization for a single station:
  - using C/A–code to broadcast clocks
  - using C/A–code to IGS products
  - using carrier–phase to IGS products
- Code–based clock comparisons used for TAI computation (baseline solutions):
  - using C/A–code in common view, single–channel receiver
  - using C/A–code in common view, multi–channel receiver
  - using P–code for GPS P3 (ionosphere–free linear combination)
- Geodetic type methods (network solutions):
  - using code and carrier phase measurements for daily independent solutions
  - using code and carrier phase measurements for a continuous solution
  - using only carrier phase measurements for a continuous solution