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Progress in Physics (50)

Formation of rogue waves under forcing fields

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The most impressive way to understand what a rogue wave is compared to ordinary waves, is to look at pictures showing a wall of water approaching ships like that in Fig. 1. Rogue waves are large amplitude waves that can appear suddenly in the open ocean, the wave crest to trough height exceeding twice the significant wave-height (defined as the mean of the upper third of the rest of the waves). Due to their large amplitude, we can easily understand how seriously these rogue waves can damage large ships or offshore structures, as reported several times in the media and in the scientific literature, and why it is important to understand how they form in order to improve the prediction of their occurrence. Rogue waves do not appear only on the ocean surface, but they are observed in many different fields of physics such as high-power pulse filamentation (Bergé *et al.* 2007), noise-induced solitons in fiber supercontinuum generation (Solli *et al.* 2007), propagation effects in optical fibers (Kibler *et al.* 2010) or in plasmas (Veldes *et al.* 2013), spatial patterns in cavities (Montina *et al.* 2009). In a broad sense, rogue waves can be defined as localised high-amplitude, statistically-rare events in a system, appearing in the tails of the associated probability distribution. In this broad sense, such huge waves are observed in many domains other than physics, such as for example financial science.



Figure 1. A wall of water approaches the Stolt Surf in October 1977. The chemical tanker encountered a wave of at least 22 m, much higher than the significant wave height estimated to 10 m. Photo credit: Karsten Petersen, www.global-mariner.com

Rare occurrence in field measurements makes the study of such phenomenon in the natural oceanic environment very difficult. A way to circumvent this problem is to use water-tank experiments, but the idealized setup of the experimental configuration limits the formation and propagation scenarios that can be analysed. Optical systems are in general more flexible and have been shown to be suitable for investigating realistic nonlinear propagation scenarios in a reproducible way (Dudley *et al.* 2014). This is an example of why the interaction between different fields in physics has been deeply developed in the last decades (see for example the ongoing ERC Multiwave project, www.ercmultiwave.eu).

Analogies in the occurrence of rogue waves in different domains have deepened our understanding of the phenomenon.

Despite this interdisciplinary effort, an explanation for the formation of rogue waves is still debated. Different possible mechanisms have been found (Kharif *et al.* 2003, 2009, Onorato *et al.* 2013), from linear processes (as for example caustic focusing in wave propagation discussed in Mathis *et al.* 2015), to nonlinear mechanisms (modulational instability), or complex dynamical processes (collisions of breathers Akhmediev *et al.* 2009 or solitons Armaroli *et al.* 2015). Interestingly, the common effort in understanding the rogue wave formation mechanisms has the consequence that many approaches are tested and shared by different communities in domains at the forefront of the research in nonlinear dynamical systems and non-equilibrium thermodynamics (see for example Hadjihosseini *et al.* 2014, Armaroli *et al.* 2015).

Starting from the Euler equations in the incompressible-fluid approximation, one can obtain propagating solutions at the ocean surface. In the deep-water limit, i.e. when the ocean depth is larger than the wave-length, these surface waves are dispersive and propagate under the effect of gravity with wave periods T in the range between 0.1 s and 5 minutes and group velocity $c_g = gT/(4\pi)$, where g is the gravitational acceleration. At leading order, i.e. by assuming narrow-banded waves of moderate amplitude that mainly propagate in one direction in a dispersive medium, the propagating waves can be modeled by the nonlinear Schrödinger equation (NLSE), written in its normalized form as follows:

$$i \frac{\partial A}{\partial t} - \frac{1}{2} \frac{\partial^2 A}{\partial x^2} - A|A|^2 = 0$$

where $A(x,t)$ is the wave envelope, and t and x are (normalized) propagation time and distance, respectively, in a frame moving at the group velocity of the input pulse. The focusing (local concentration of energy) is obtained by the mutual effect of dispersion (the second term) and nonlinearity (the last term in the above equation).

Exact solutions of the NLSE, such as the Akhmediev breather (Akhmediev & Komeev 1986) or the Peregrine soliton (Peregrine 1983), play a crucial role in the formation of rogue wave (see Fig. 2): they are supposed to form the deterministic backbone of nonlinear focusing of wave energy in a background wave state.

However, the nonlinear evolution implies a broadening of the spectrum and a steepening of the waves, and suddenly brings the system in a regime that can no longer be described by the NLSE. Higher-order effects need to be included (higher-order dispersions and nonlinearities, described by more general models, e. g. Dysthe 1979 and Ankiewicz *et al.* 2013). Within these generalized models,

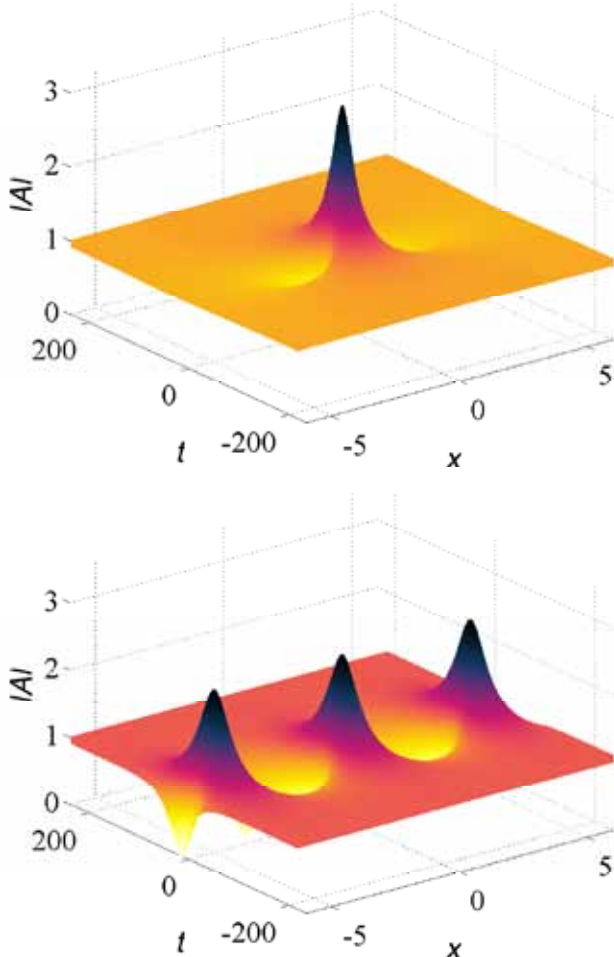


Figure 2. Exact solutions of the NLSE. Top: Peregrine soliton. Bottom: Akhmediev breather.

nonlinear superposition of first-order solutions (such as the Peregrine soliton or the Akhmediev breather) are able to form rogue waves with amplitudes much larger than the constituent breathers, as shown in numerical and laboratory experiments (Akhmediev *et al.* 2009, Erkintalo *et al.* 2011, Armaroli *et al.* 2015).

The evolution and the formation of rogue waves are also affected by dissipative effects (Segur *et al.* 2005) and external forcing of different origin (wind for ocean waves, Raman scattering in optical systems, etc.) that need to be included in the models. The complexity of this picture explains why a unified approach that describes the formation of rogue waves is still missing and why a huge effort in sharing methods and ideas is under way in the scientific community.

Since ocean waves are under the continuous action of wind, it is worth asking how the constituent breathers modify in a forcing regime. It is known that breather solutions, that undergo periodic energy exchange with a finite background, correspond to the evolution of the modulational instability, a central process of physical systems described by the NLSE (Akhmediev & Korneev 1986, Zakharov & Ostrovsky 2009). Thus the question is equivalent to asking how the modulational instability is modified in a forced regime.

The wind can either damp the ocean wave amplitude when it blows slower or opposite to the propagation direction, or pump energy into the waves. Many experiments have been performed to investigate how surface waves and modulational instability are affected by wind and dissipation (Bliven *et al.* 1986, Waseda & Tulin 1999, Segur *et al.* 2005, Grare

et al. 2013, Chabchoub *et al.* 2013) but the results are often in contradiction due to the difficulty of reducing the complexity of the interacting processes to simplified configurations. One can start by considering the simple framework where a quasi-laminar air flow interacts with a wave group that evolves under potential flow approximation. In this case the Miles mechanism provides the basis for energy exchange between wind and waves and the hydrodynamics equations can be handled analytically.

As in many complex systems, ocean waves are described as multi-scale processes. Thus the multi-scale method, where temporal and spatial scales for the envelope can be separated from the much smaller carrier-wave scales, can be used to introduce forcing and wave effects at the proper order (Brunetti *et al.* 2014). The main parameters are the steepness of the wave, ε , which is a measure of the degree of nonlinearity of the problem, and the Miles growth rate, Γ , which depends on the wind strength. The resulting forced NLSE has the following form (in normalized quantities, see Brunetti & Kasparian 2014):

$$i\frac{\partial A}{\partial t} - \frac{1}{2}\frac{\partial^2 A}{\partial x^2} - A|A|^2 = i\Gamma A + 3\Gamma\frac{\partial A}{\partial x} + \frac{1}{2}\Gamma^2 A$$

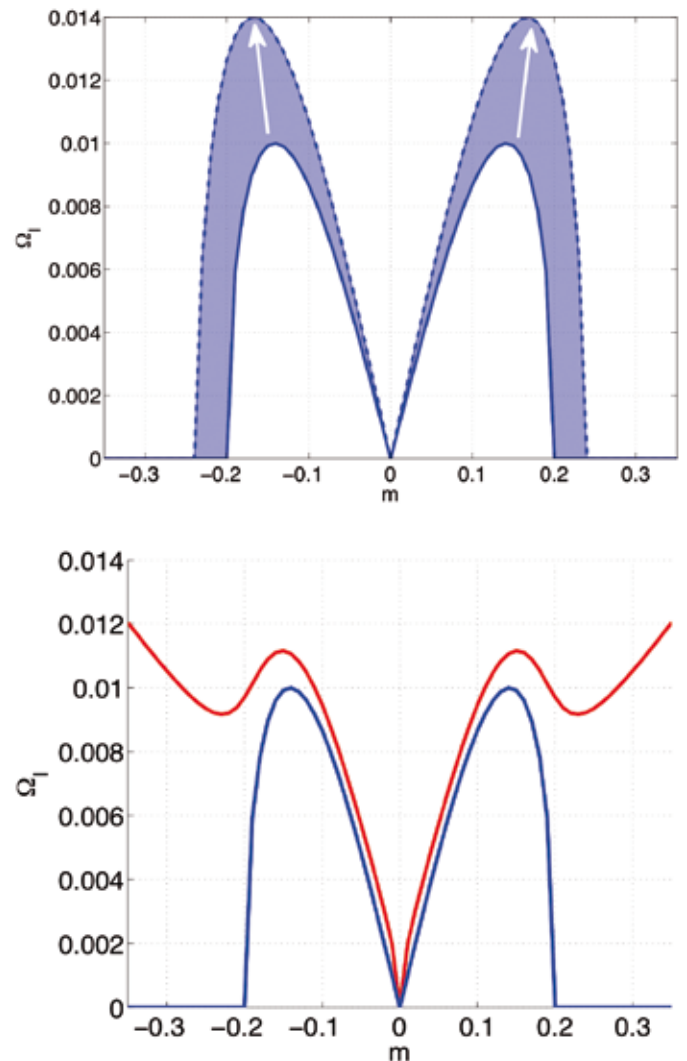


Figure 3. Effect of wind forcing on the gain band of the modulational instability. Top: in the case of slow wind (i.e. the (normalized) Miles growth rate Γ is much smaller than the wave steepness ε), the range of modulational wavenumbers m under the positive-gain band Ω_1 increases in time. Bottom: in the case of strong wind, $\Gamma \sim \varepsilon$, the gain band is enhanced (red curve) with respect to the case without wind (blue curve) from the beginning.

It can be shown that the effect of the first term on the right-hand side is to increase in time the coefficient in front of the nonlinear term (Proment & Onorato 2012), while the effect of the last two terms on the right-hand side is to modify the dispersion term (Brunetti et al. 2014), thus wind affects the focusing process. The last two terms on the right-hand side disappear in the case of low Miles growth rates. In this regime of slow wind, the region of instability enlarges in time (Leblanc 2007), as shown by the shaded blue area in Fig. 3a. As a consequence, the range of modulational wavenumbers m within the positive-gain band Ω_1 increases in time. In the regime of strong wind, on the contrary, where the last two terms become dominant, the modulational instability is enhanced from the beginning, as shown by the position of the red curve in Fig. 3b (with respect to the blue curve which corresponds to the unforced regime). In this case, the gain band of the modulational instability has infinite width and thus induces a more important broadening of the initial spectrum with respect to the case of slow wind. This is a consequence that can be tested in air-sea interaction facilities such that installed at Pythéas-IRPHE/Luminy, Marseille (France), shown in Fig. 4.



Figure 4. Wind-wave facility at Pythéas-IRPHE in Marseille (France). Photo credit: Arthur Lemoine. The tank is 40 m long, 3 m wide and 0.9 m deep. It is equipped with a recirculating wind tunnel that can generate wind speeds between 1 and 14 m/s.

Experiments have been carried out this summer in the French facility, and preliminary results (Eeltink *et al.*, in prep.) show a series of interesting phenomena that need further investigation. The wind affects the energy spectrum in an asymmetric way and induces downshifting, i.e. the smaller sideband grows faster than the larger one. Downshifting can be also produced by dissipative effects (as wave breaking) or higher-order nonlinearities (because of the presence of odd-derivative terms, as in the Dysthe model). The difficulty in this kind of experiments is to find regimes where one process predominates, since due to the nonlinear dynamics of such systems it is not possible to completely isolate one process from the other. Numerical simulations and analogies between different domains are crucial to gain insight into these complex questions.

Acknowledgements

This work was partially supported by the Swiss National Science Foundation.

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