Nr. 69, Februar 2023

SPG Mitteilungen
Communications de la SSP

Auszug - Extrait

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DOI: *10.5281/zenodo.8025731*

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What is music? Since the answer depends on whom you ask, I feel free to provide my favorite definition: The core elements have to do with dancing (rhythm) and singing (melody). Music is rhythm, melody, or a combination of these two. There exist innumerable refinements involving polyphony, harmony, tonality, instrumentation, structure. Humans can make music, animals too, so do natural phenomena and inanimate objects. Certain aspects involve human creativity or – for the spiritually inclined among us – divine inspiration, for other aspects we all agree that these follow the laws of physics. Obviously, the latter is the core material of courses on physics of music [1,2]. However, the former is what beauty is about, and it would be a sin to omit it. This makes preparing and giving courses in physics of music such a great adventure. The subject invites teachers and students to delve in a goldmine of subjects involving history, linguistics, religion, music, anatomy and physics.

I start by summarizing a few essential points about music and tonal intervals. Melodies sung or played on an instrument are sequences of tones distributed along a musical scale. An example of such a scale is the sequence of 8 white keys on a piano keyboard starting at the central **c**: **c-d-e-f-g-a-b-C**. The 8th tone (**C**) has twice the frequency of the first one (**c**), and after the **C** the series continues with the other – also frequency doubled – tones. Not all these tone intervals are equal: The frequency ratios **d:c**, **e:d**, **g:f**, **a:g** and **b:a** are approximately 1.12 (a "whole-tone interval"), whereas **f:e** and **C:b** are approximately 1.06 (a "semitone interval"). Since two different tonal intervals are used, the scale is called "diatonic", and in this example the sequence of intervals is 1.12, 1.12, 1.06, 1.12, 1,12, 1,12, 1.06. We call this sequence the "Ionian mode". If we start at the **a**, and we play again only the white keys, the sequence of intervals becomes 1.12, 1.06, 1.12, 1.12, 1.06, 1.12, 1.12. We call the latter sequence the "Aeolian mode". We see that the pattern of intervals is different from the Ionian mode. Of course, if we sing, we are not confined to the tones imposed by the keys of a piano, and thus we could equally well sing the Aeolian mode starting from any other tone than the **a**, including the **c**. Since the Ionian and Aeolian sequences differ from each other, we now have 2 sequences starting from the **c**, each sounding differently. As a next step we could adopt a different order of the notes, for example we play or sing something like **g-g-g-C-C-b-b-g-g-g-D-D-C-C**. These are the tones of the first strophe of a sunny song that some of you will recognize. If we transpose it down in the same way as we did for the scale of **c**, and once again play only the white keys, we get **e-e-e-a-a-g-g-e-e-e-b-b-a-a**, sounding a little less sunny, let's say a bit more melancholic. These two different ways to set this melody represent two different "modes" as it is called in music theory. Modes have been used since antiquity: heptatonic modes such as the two examples above, and also pentatonic modes. Mesopotamians distinguished in the beginning of the second millennium BCE seven heptatonic modes [3], which they called *Išartu*, *Kitmu*, *Embūbu*, *Pūtu*, *Nīd qabli*, *Nīš gabarî* and *Qablītu* [4]. The ancient Greeks rebaptized this using the names of the regions where these modes were most commonly used, from which we inherited the denominations Ionian, Dorian, Phrygian, Lydian, Mixolydian, Aeolian and Loerian that are still in use today. Regardless as to whether your taste is for classical, religious or popular music, if you regularly listen to music or play or sing it yourself, you are frequently exposed to any one of these seven modes.

In the above description I gave the values ~1.06 and ~1.12, which are only approximate. In practice one doesn't sing these intervals. A singer automatically tunes to natural intervals, i.e. (s)he adjusts one frequency (f_1) to another one (f_2) such that the ratio $f_1 : f_2 = h : j$ where h and j are integer numbers. How come? Superimposing the sound of two frequencies produces beats of the sound amplitude, beating at the difference frequency. Most humans perceive a beat frequency higher than 2 to 4 per second as unpleasant, and a slower beating as acceptable. A natural consequence is that musicians tune the frequencies they sing such, as to eliminate beating as much as they can. Comparing two frequencies, this requires that their ratio is given by $h : j$ where h and j are integers. This practice is called "just intonation". If people sing different musical lines at the same time while respecting just intonation, the harmonies resulting from this polyphony are very pleasing to the ear. Depending on the structure of the melody there may be some peculiar consequences. The perfect harmonies force the tuning in the course of certain sequences to increase or decrease by one or several tiny intervals known as "syntonic commas" (about ¼ of a chromatic semitone interval) first described by ***Didymus the Musician*** in the first century CE [5]. ***Giambattista Benedetti*** (1530 - 1590), a mathematician with an interest in physics and the science of music, wrote a musical composition to demonstrate this tonal migration effect [6]. ***Michaelangelo Rossi*** (1601 - 1656), composer, violinist and organist recognized that tonal migration is a real problem for an ensemble of good singers using just intonation. Performing his madrigal "Per non mi dir" from beginning to end one accumulates an overall migration of 11 syntonic commas (about 3 semitones!). Rossi organized courses specifically to instruct choirs how to avoid this problem. A very instructive discussion about this is offered by ***Elam Rotem*** – contemporary composer, singer and harpsichord player based in Basel – on his youtube channel Early Music Sources [7].

Among physicists one finds – just as in any other sector of society – music lovers: people who play an instrument, who sing, who have studied music on some level, from elementary to professional. The possession of an analytically gifted mind does not exclude having an artistic side as well. Good scientists are creative, and creativity finds many different outlets. In the history of science, one finds many scientists who had a dual career in science and music. To illustrate this, I will describe below how the physics of music has evolved throughout the history of mankind.

The first scientific research on this subject is traditionally attributed to ***Pythagoras*** (570 - 495 BCE). Pythagoras used a monochord, an instrument already mentioned in Sumerian texts, to establish that the ratio 1:2 of the wavelength of a vibrating string corresponds to an octave (in modern termi-

Figure 1. Pythagoras doing experiments with various different instruments. Woodcut [8].

nology), and the ratios 3:2 and 4:3 to a fifth and a fourth respectively [8] (see Fig. 1). ***Plato*** (423 - 348 BCE) appeals in the Socratic dialogue Timaeus to the notions of ἐπόγδοον (“greater by an eighth part”) and λεῖμμα (“residue”). The former he identified as the ratio 9:8, an interval which we call nowadays “major second”, the latter as 256:243, corresponding to “minor second” in contemporary terminology. Much later ***Marin Mersenne*** (1588 - 1648), ordained Catholic priest and polymath, developed the laws describing the harmonics of a vibrating string. His seminal work on music theory, “Harmonie universelle”, earned him the epithet "father of acoustics". Returning to antiquity ***Aristoxenos*** (360 - 300 BCE) treated in “Aristoxenou Harmonika Stoicheia” the nature of musical intervals and scales. Unlike Pythagoras, Plato and Aristotle, who considered music in the context of cosmology and ethics, Aristoxenus examined the tonal structure of music as a system in itself and studied the relationship between notes “as multiples of a particular unit of measurement… for example, the interval of a fifth as three and a half tones without reference to the ratio of 3:2 and in this way it is more compatible with the way that the human mind perceives music.” [9] ***Klaudios Ptolemeos*** from Alexandria (100 - 170) based tonal intervals on arithmetic ratios backed up by empirical observation, and he proposed various different ways to divide the octave. His book “Harmonikon” where he described all this – and more – had quite an impact: Even fourteen centuries later ***Gioseffo Zarlino*** (1517 - 1590) considered Ptolemy's “intense diatonic scale” the only tuning that could be reasonably sung. ***Christiaan Huygens*** (1629 - 1695) applied logarithms, right after the mathematics of logarithms had been introduced, to advance the understanding of musical scales and for the development of equal temperament. ***Johannes Kepler***, ***Nicolaus Mercator***, ***Isaac Newton***, ***René Descartes***, ***Jost Bürgi*** all worked on the division of the octave [10,11]. Descartes and Bürgi used diagrams bearing a striking resemblance to the “volvelles” (circular wheel charts) that had been used by astronomers since the Middle Ages. To be specific, they used the circle as a metaphor for the octave in combination with a logarithmic representation of musical ratios [12]. ***Leonhard Euler*** (1707 - 1783) described in “Tentamen novae theoriae musicae” [13] a conceptual lattice diagram representing tonal space (the “Speculum musicum”, see Fig. 2), which in the 19th century drew renewed interest as the “Tonnetz” in neo-Riemannian mathematics of music (after ***Hugo Riemann***, not to be confused with the more famous Bernhard Riemann). ***Thomas Young*** (1773 - 1829) “the last man who knew everything” busied himself with inventing the double-slit experiment named after him, Young's modulus, astigmatism, functioning of heart and arteries, a grammatical comparison and vocabulary of 400 languages, and deciphering Egyptian hieroglyphs, in particular the Rosetta stone. He also invented a temperament, i.e. a subdivision of the octave, that has been used and appreciated by composers and musicians throughout the 19th century. In the beginning of the 20th century Young’s temperament got – much undeservedly – abandoned under the influence of the mass production of pianos, and ever since mankind has been systematically brainwashed with equal temperament [14,15]. ***Hermann von Helmholtz*** (1821 - 1894) developed a mathematical theory to explain the timbre by overtones. He described this theory in “Die Lehre von den Tonempfindungen als Physiologische Grundlage für die Theorie der Musik”, a classic in the literature of science of music [16]. ***Adriaan Fokker*** (1887 - 1972) was a theoretical physicist. He also introduced a new method (the “Fokker periodicity block”) to relate musical intervals in just intonation to those in equal tuning, designed and built keyboard instruments capable of playing microtonal scales via a generalized keyboard, among which an organ which has 31 tones in the octave for which he also composed music.

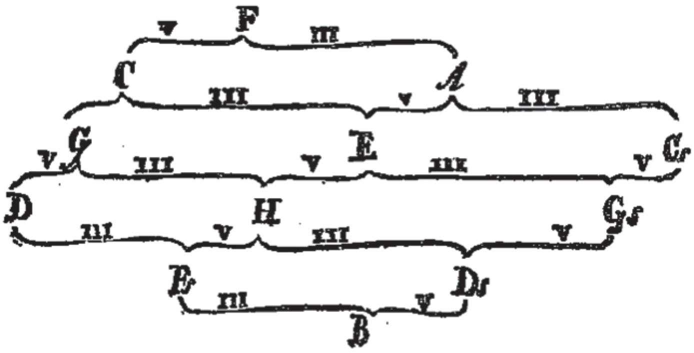

Figure 2. Leonhard Euler's Tonnetz [10] showing the triadic relationships between the perfect fifth, major third and minor third. The ***f*** *is on top, left below it the* ***c*** *(****c:f*** *= 3:2, a perfect fifth), and to the right the* ***a*** *(****a:f*** *= 5:4, a major third). The ratio* ***a:c*** *is then 5:6, corresponding to a minor third. These arithmetic relations are repeated along the vertical and horizontal directions of the graph.*

Why should we care about the interface of music and science? Students. They are openminded, they are curious. And they may, just may, get the totally peculiar impression that physics is only about equations and numbers. Which might be perceived by some of these inquisitive minds as boring, as unjustified as that may sound to matured physicists such as many of us are. And unjustified this impression is. Many of us have the privilege of doing curiosity driven research, and usually we find our own most recent scientific result the most interesting of all. Editorial boards of high-profile journals bear the burden of our enthusiasm in the form of an incessant deluge of submissions reporting – as it was so aptly formulated by the late professor ***Cor Haas*** (1930 - 2019) – "a negligible correction to an unobservable effect". However, if you place yourself in the position of an adolescent, would you be really attracted by the scientific research that you are presently doing? For certain subjects it may be love at first sight, but we shouldn't forget that for many of the subjects on which we presently work it took us some time to perceive the beauty of it.

That said, the subject of physics of music appeals quite readily to peoples' imagination. How do I know? Experience. I give two of the many possible examples:

Example 1. This morning (29 December) I interrupted my frantic writing of this article to beat the 31 December deadline for a cup of coffee, and looked at today's newspaper. There it was: A photo filling half the frontpage with the header: Arithmetique de l'accoustique, accompanied by a long and interesting article on page 9 explaining that two tones played simultaneously on a violin (called "dyade") produce a difference tone that is audible to the human ear, also of non-experts. The effect is quite distinct in ancient instruments; the Italian composer ***Giuseppe Tartini*** discussed it in 1714. This effect is not noticeable in modern violins [17,18].

Example 2. In the past 5 years I've given modules of "physique de la musique" as part of a full-year course "physique du quotidien" of my colleague Andreas Müller at the Department of Physics of the University of Geneva [19]. In 2021 this was attended by an auditor, Eric Rey, who contacted me after my course with the proposal to mount a public lecture together with him and the other four members of Geneva Brass. I said "yes", worked hard, and so did the 5 professional musicians of Geneva Brass. That said, we weren't so sure if anyone would show up for our conference-concert "la Physique Résonne" on a rainy evening in early November [20]. The auditorium had place for 300. It turned out to be not big enough, people were sitting on the stairs. During 90 minutes musical interventions and demonstrations played by the brass quintet alternated with explanations by me on questions such as: What is a resonance? How does sound travel through air? How is the sound produced by a wind instrument defined by its shape? What makes a chord consonant or dissonant? Why can a piano or an organ only be out of tune? Is the pitch of the alphorn higher or lower at the top of the Matterhorn than down below in the valley? (see Fig. 3) And how does that change at the North Pole, in the Sahara, or on Mars? The audience, consisting of non-experts in physics and music, loved it. Why? My understanding is, that the subject appeals to seemingly antagonistic aspects of the human mind, namely the analytical and the artistic, and that is intriguing.

Figure 3. Christophe Sturzenegger (Geneva Brass) playing alphorn at the summit of Dent Blanche (left, 4356 m) and in Crans-Montana (right, 1495 m)

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