Lemaître's inhomogeneous cosmological model of 1933

and

its recent revival

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Bern, November 21, 2019

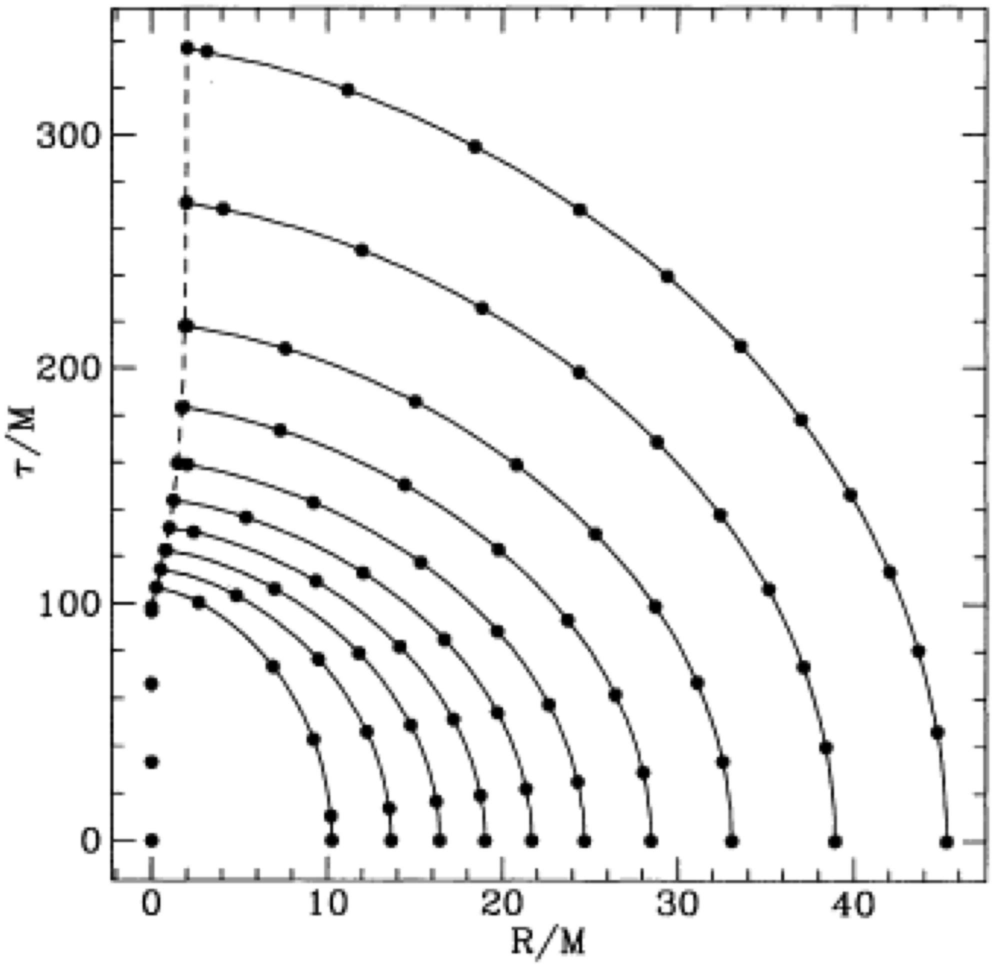
Original publication:

L'Univers en expansion, Annales de la Société Scientifique de Bruxelles A 53 (1933), 51-85.

About 20 years ago it was printed in English translation:

The Expanding Universe, General Relativity and Gravitation, Vol.29, No.5 (1997).

Gravitational Collapse for "Tolman-Bondi Model", ApJ, (1995)!



Lemaître shows that the "Schwarzschild singularity" is apparent

In late Sect.11 of this paper, which deviates from the main theme, entitled: **SCHWARZSCHILD'S EXTERIOR FIELD**.

This most remarkable contribution has been overlooked by almost all researchers (including Einstein) in the field.

Lemaître shows in this section that the so-called Schwarzschild singularity, that disturbed relativists over decades, is spurious. In his words:

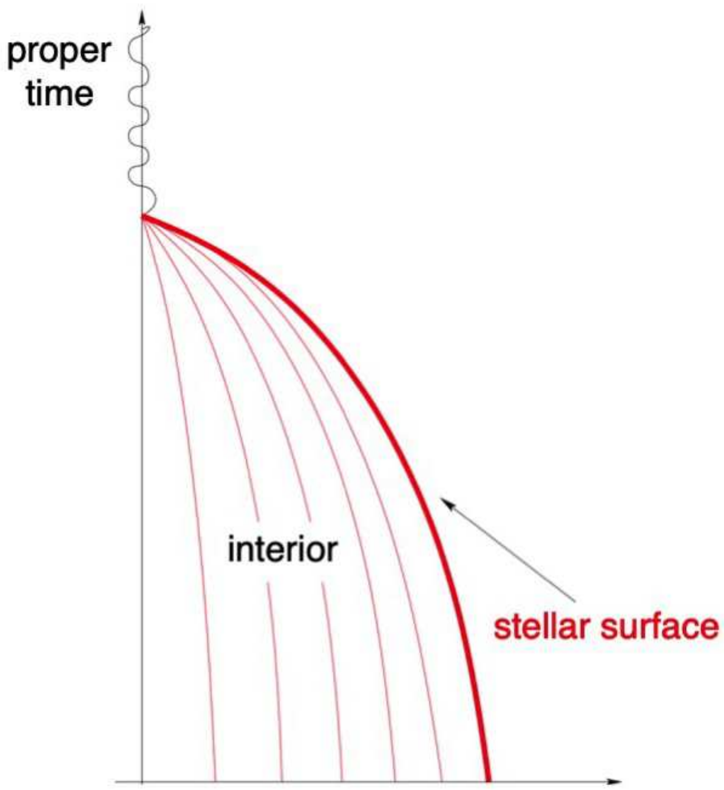
"The singularity of the Schwarzschild field is thus a fictitious singularity".

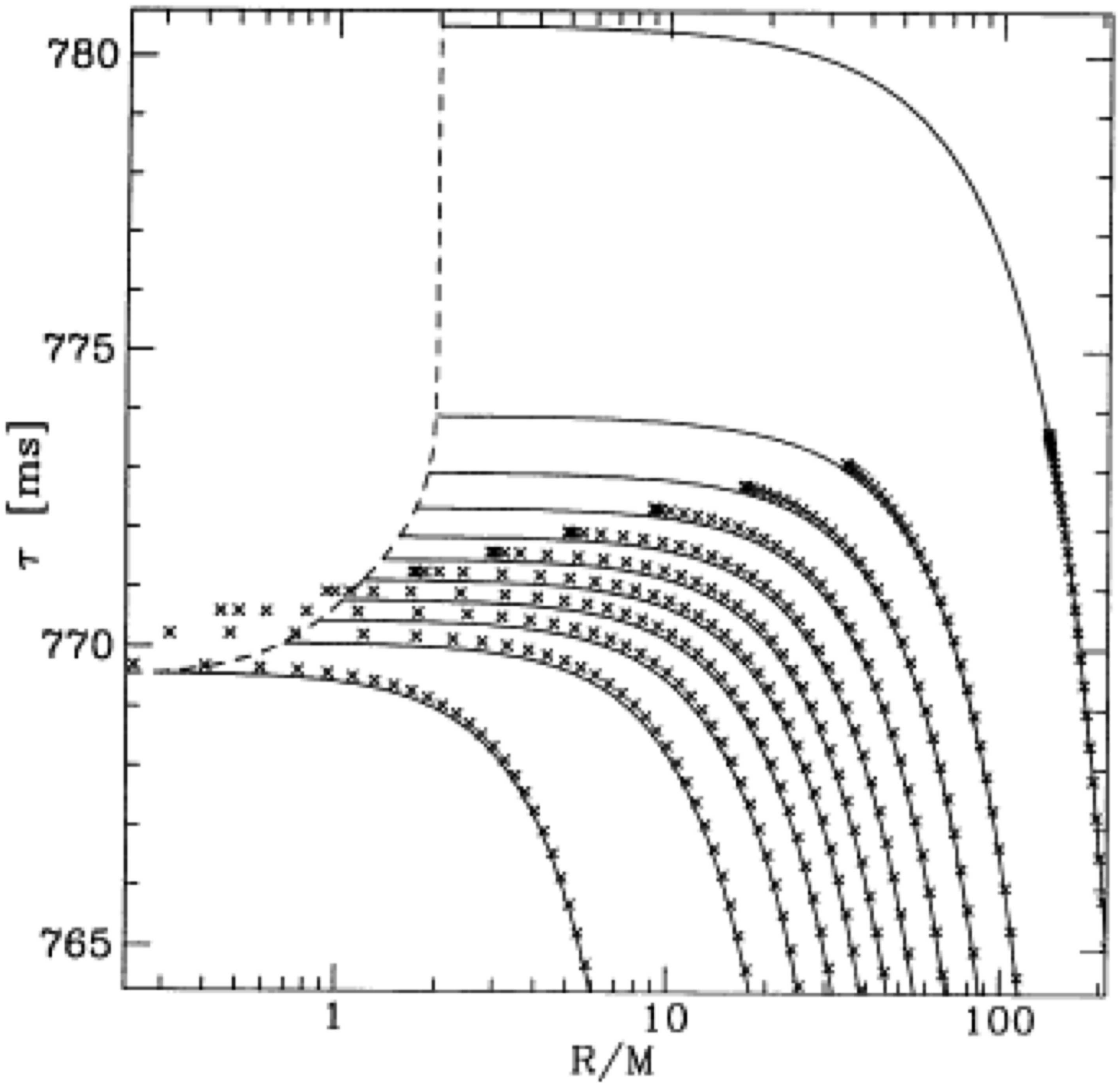
$$ds^2 = -c^2\left(1 - \frac{2m}{r}\right)dt^2 + \frac{dr^2}{1 - 2m/r} + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)$$

"La singularité du champs de Schwarzschild est donc une singularité fictive, analogue celle qui se présentait a l'horizon du centre dans la forme originale de l'universe de de Sitter".

J. Robert Oppenheimer and his student Hartland Snyder in 1939 on the gravitational collapse of a simplified stellar model (special application of Lemaître's equations, did not understand the "Schwarzschild singularity"). Was only understood after Einstein's death.

Lemaître: spherically symmetric inhomogeneous dust models for collapsing stars or the expanding universe; C.W. Misner, D.H.Sharp (1964): straightforward extension to $p \neq 0 \rightarrow$ num. sim. in 1980s.





780
775
770
765
τ [ms]
1
10
100
R/M

Recent confrontation with observations and results of the LTB model

Since the required magnitude of dark energy in ΛCDM models is a mystery, a minority of cosmologists has in recent years investigated the possibility that the observational findings might be caused by inhomogeneities in the distribution of matter and other quantities.

$\rightarrow$ revival of the LTB model: Do we live in an underdense region of the universe centered not far from us, and do not need dark energy? (Example for nonlinear perturbations of FL-models; more general ones for the future, using the tools of numerical relativity.)

Nicholas of Cusa in the fifteenth century in "**Of Learned Ignorance"**:

The Universe "is a sphere of which the center is everywhere and the circumference is nowhere".

Giordano Bruno: "Im Universum gibt es keien Mittelpunkt un keine Peripherie, sondern der Mittelpukt ist überall."

The model is determined by the **matter density** $\rho(t_0, r)$ at the present time (plus some cosm. param.). Constraints by observational data?

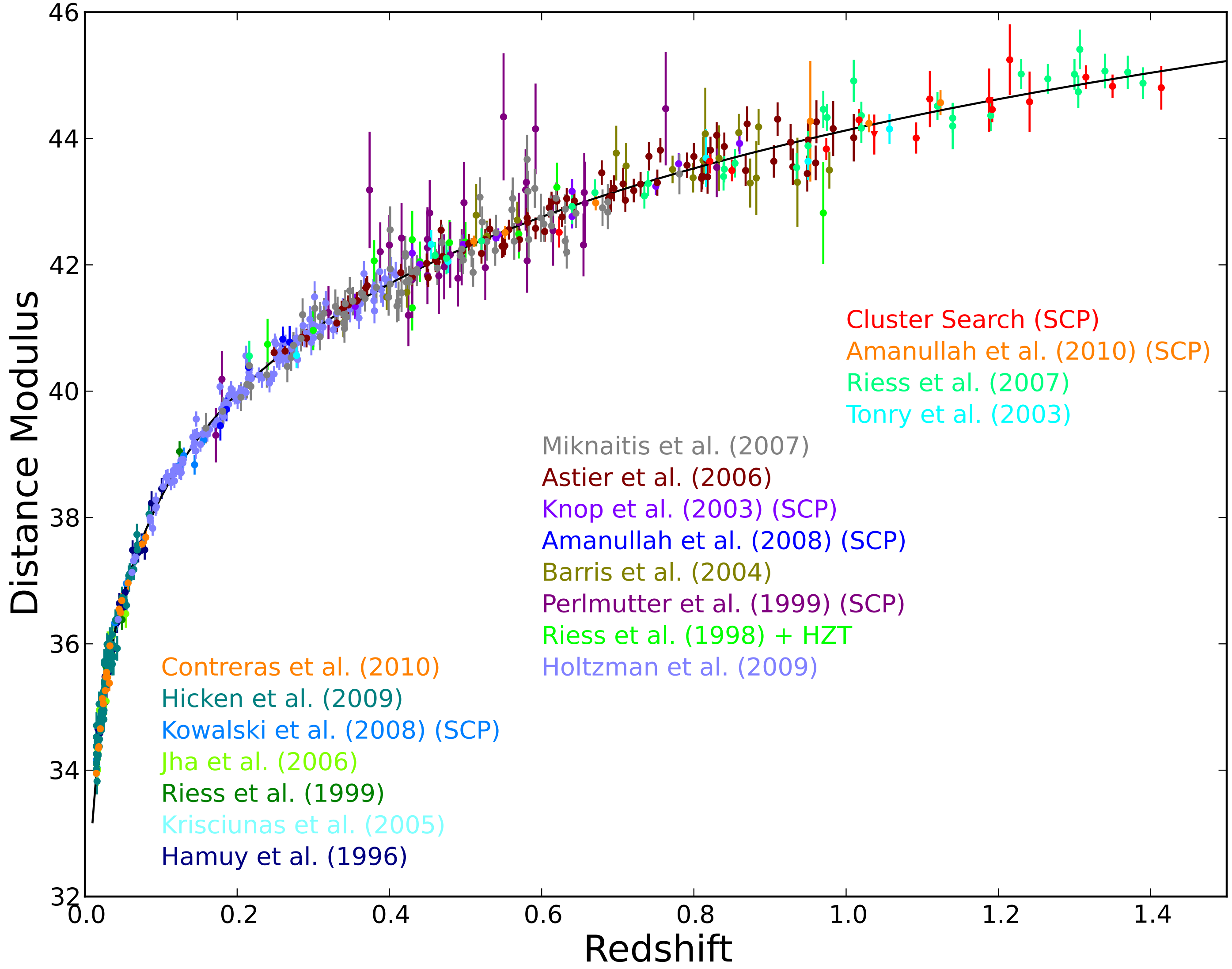
summarize paper: Redlich, M., et al. 2014, A&A, 570, A63

assumption: The early Universe was homogeneous until the time of recombination and followed standard physics.
used observational data:

- measured **local Hubble rate**: $H_0 \simeq 74$ km s^{-1} Mpc^{-1};

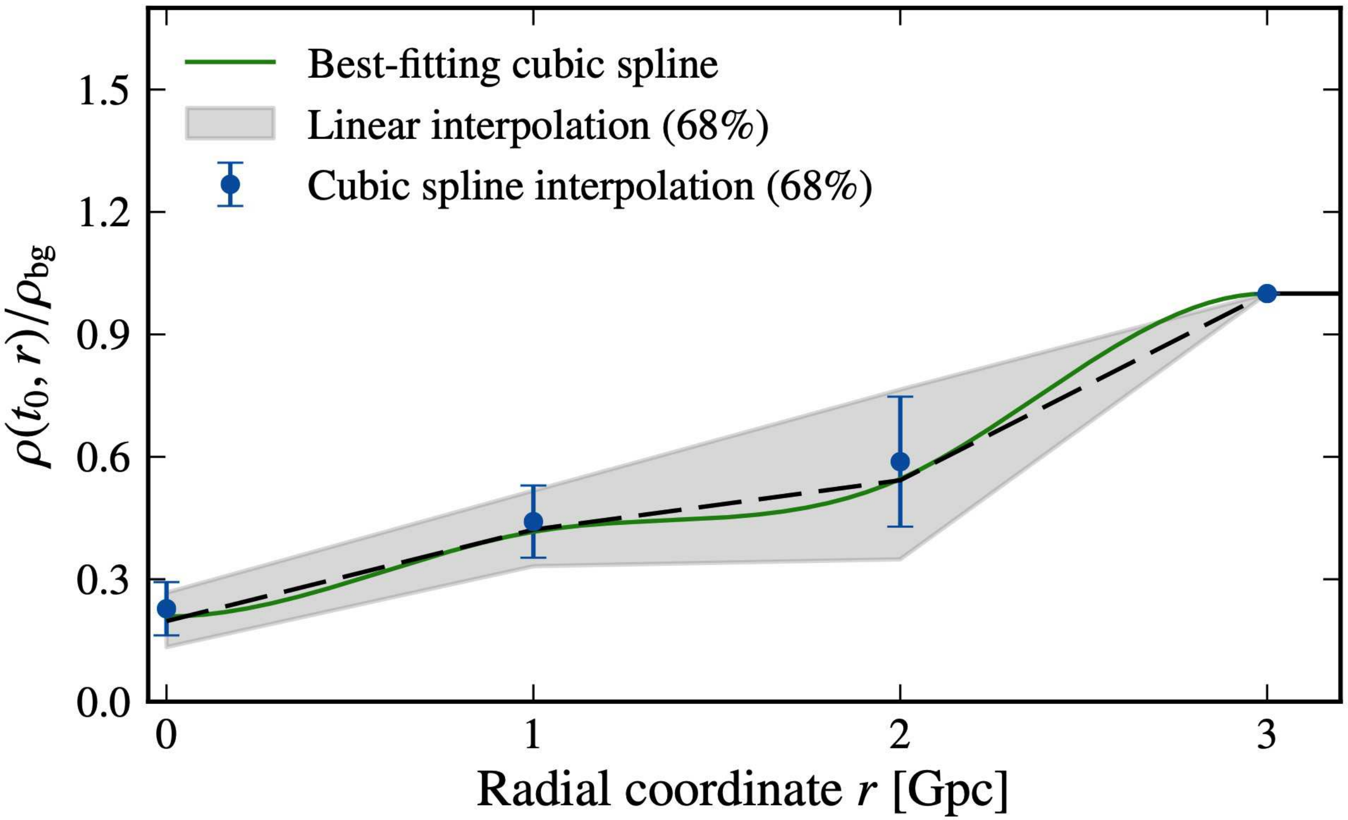
- **supernovae data**;

- **angular diameter distance to the surface of last scattering**: $D_A = 12.80 \pm 0.068$ Mpc from *Planck* data (Vonlanten, Räsänen & Durrer; Audren).



46
44
42
40
38
36
34
32
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Distance Modulus
Redshift
Cluster Search (SCP)
Amanullah et al. (2010) (SCP)
Riess et al. (2007)
Tonry et al. (2003)
Miknaitis et al. (2007)
Astier et al. (2006)
Knop et al. (2003) (SCP)
Amanullah et al. (2008) (SCP)
Barris et al. (2004)
Perlmutter et al. (1999) (SCP)
Riess et al. (1998) + HZT
Holtzman et al. (2009)
Contreras et al. (2010)
Hicken et al. (2009)
Kowalski et al. (2008) (SCP)
Jha et al. (2006)
Riess et al. (1999)
Krisciunas et al. (2005)
Hamuy et al. (1996)

Conclusions: (i) The local Hubble rate + supernovae data can easily be fitted (for $\Lambda = 0$); data favor formation of **large and deep voids**:



(ii) "Model-independent" constraints from *Planck* + supernovae data imply an **unrealistically low value of the local Hubble rate**, $H_0 \approx 39$ km s^{-1} Mpc^{-1} .

Quote: *"LTB models with a constant bang function and zero cosmological constant are inconsistent with current data."*

The final sections are devoted to LTB models with a *non-vanishing cosmological constant*.

Lemaître's view of Λ, expressed again in his contribution to the famous Schilpp volume "Albert Einstein: Philosopher-Scientist, entitled: The Cosmological Constant.

Even if the introduction of the cosmological constant "has its sole original justification, that of leading to a natural solution of the cosmological problem" (Einstein), it remains true that Einstein has shown that the structure of his equations quit naturally allows for the presence of a second constant beside the gravitational one. This raises a problem and opens possibilities which deserve careful discussion. The history of science provides man instances of discoveries which have been made for reasons which are no longer considered satisfactory. It may be that the discovery of the cosmological constant is such a case."

Lemaître on vacuum energy (Nature **127**, 706 (1931); Gen. Rel. Grav. **43**, 2911 (2011)):

"In order that motion relative to vacuum may not be detected, we must associate a pressure $p = -\rho c^2$ to the density of energy ρc^2 of vacuum ... according to $\rho = \Lambda c^2/4\pi G \sim 10^{-27} gcm^{-3}$."

- Rediscovered by *Sakharov* in 1967

Einstein's Conversion and rejection of the Λ-term

Einstein. A. (1931). Sitzungsber. Preuss. Akad. Wiss. 235-37.

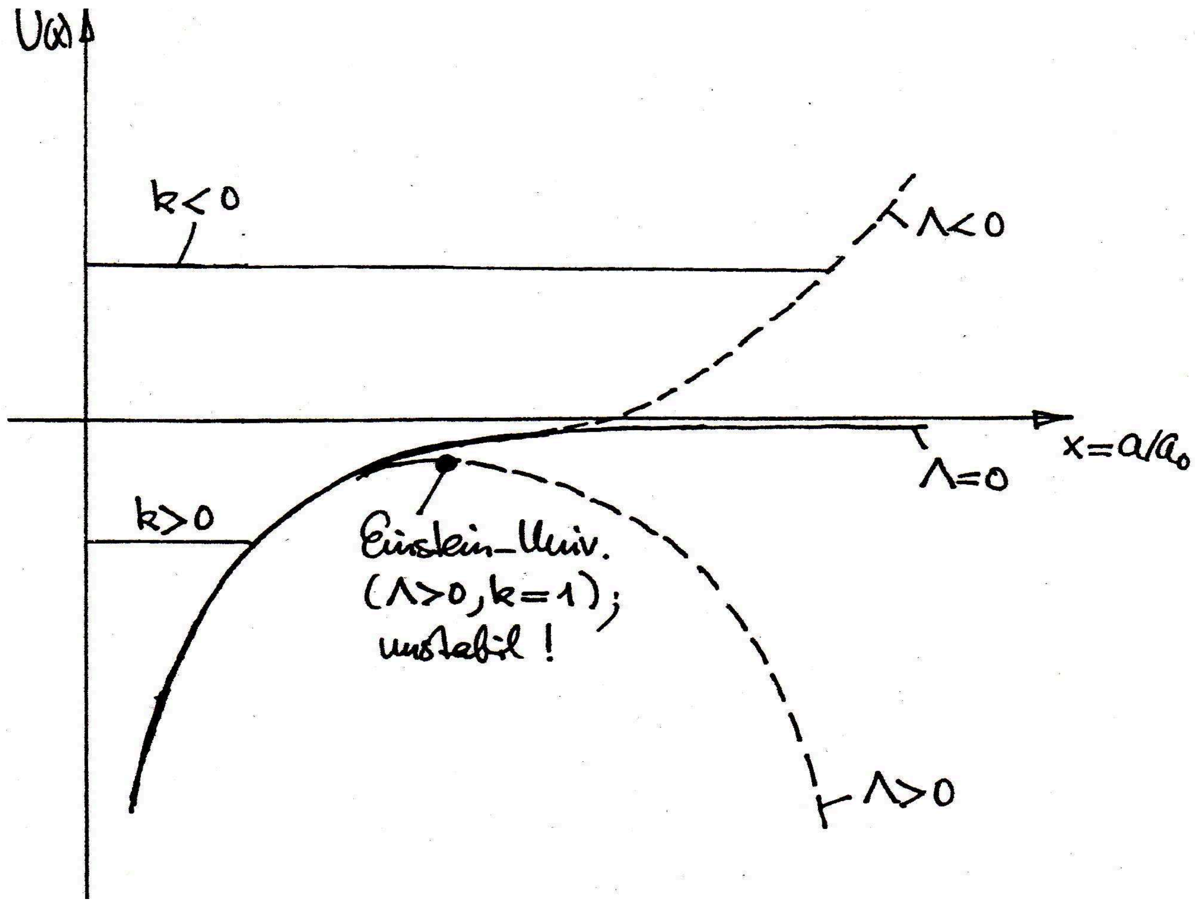
Citations:

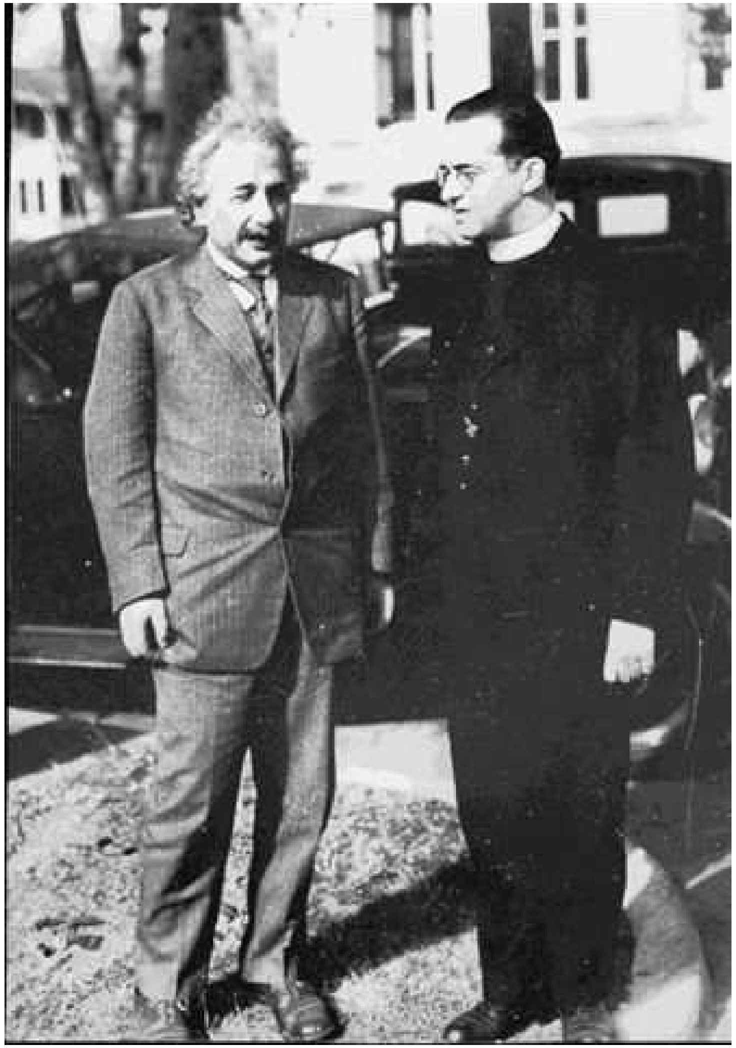
A. Einstein. 1931. Sitzsber. Preuss. Akad. Wiss. ...
A. Einstein. Sitzber. Preuss. Akad. Wiss. ... (1931)
A. Einstein (1931). Sber. preuss. Akad. Wiss. ...
Einstein. A .. 1931. Sb. Preuss. Akad. Wiss. ...
A. Einstein. S.-B. Preuss. Akad. Wis. ...1931
A. Einstein. S.B. Preuss. Akad. Wiss. (1931) ...

Einstein, A., and Preuss, S.B. (1931). Akad. Wiss. **235**.

(*"General Relativity and Gravitation"*, One hundred years after the birth of Einstein, Vol.2, pp. 329-357.)

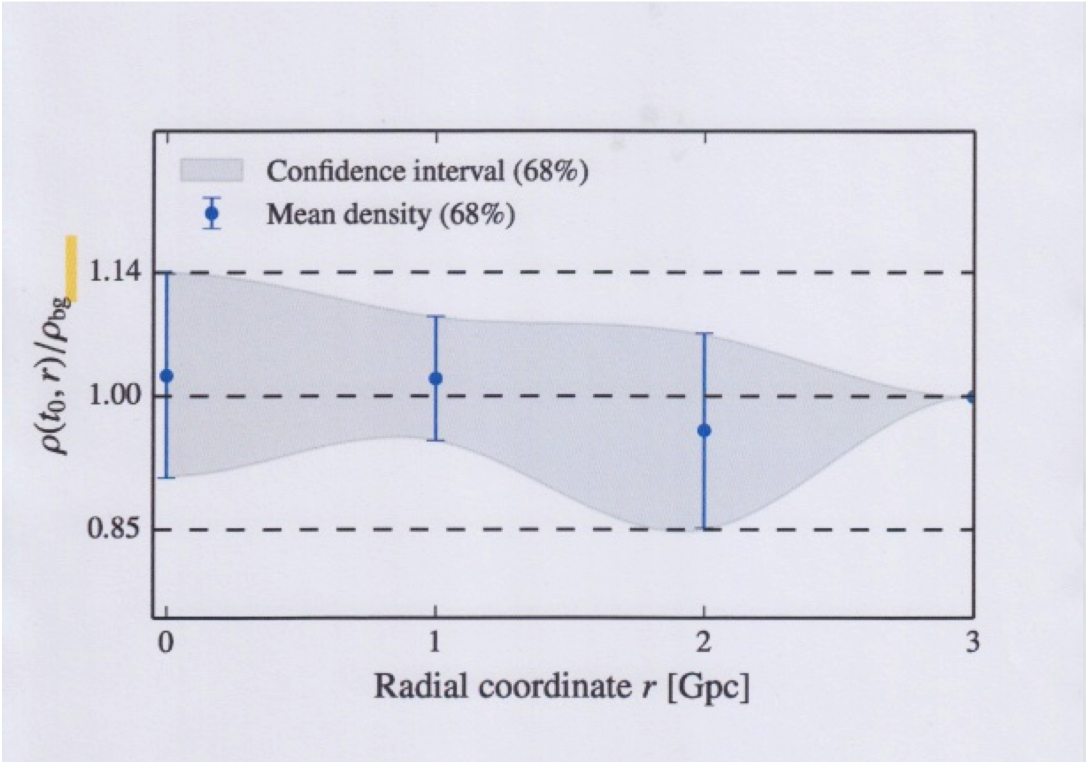
Lemaître's hesitation model (1927) and instability of Einstein universe; effective potential $U_\Lambda(x)$ of mechanical interpretation:



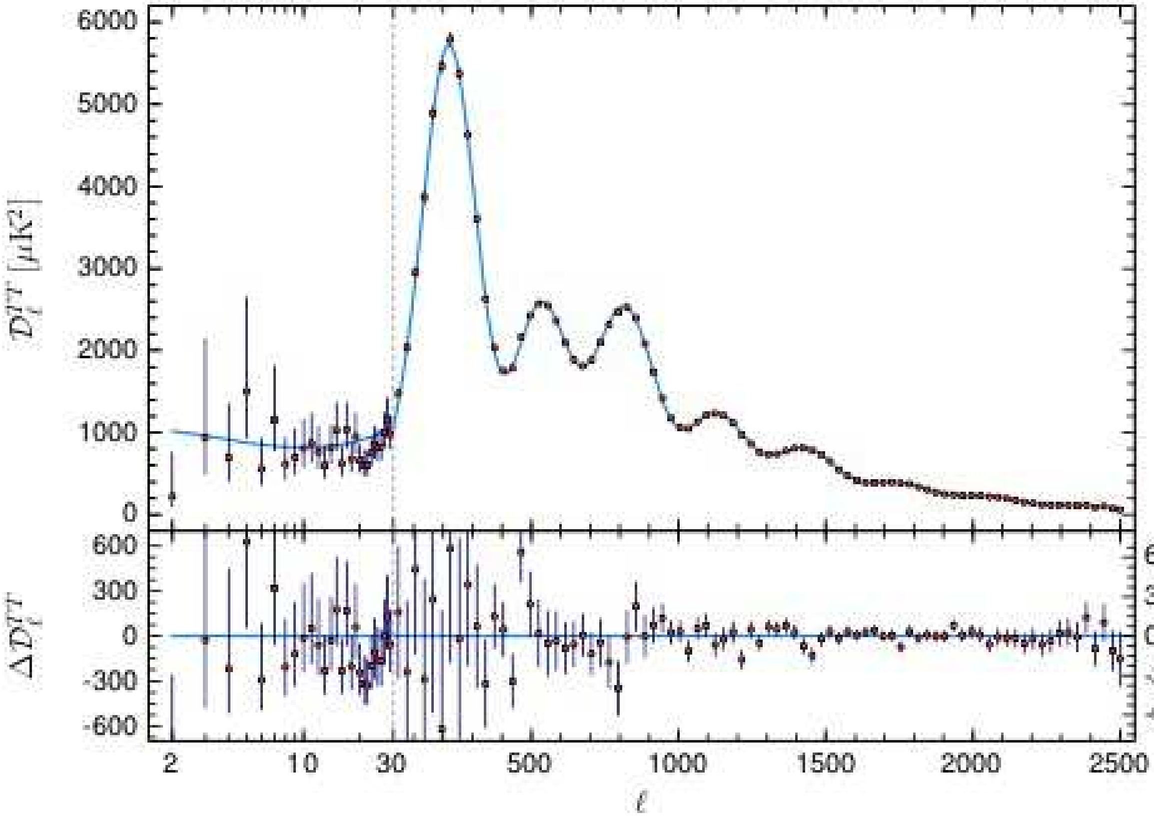


The authors arrived with their detailed analysis at the result, that the data are fitted better than for the FL model (with Λ), but that the improvement is "*almost negligible*".

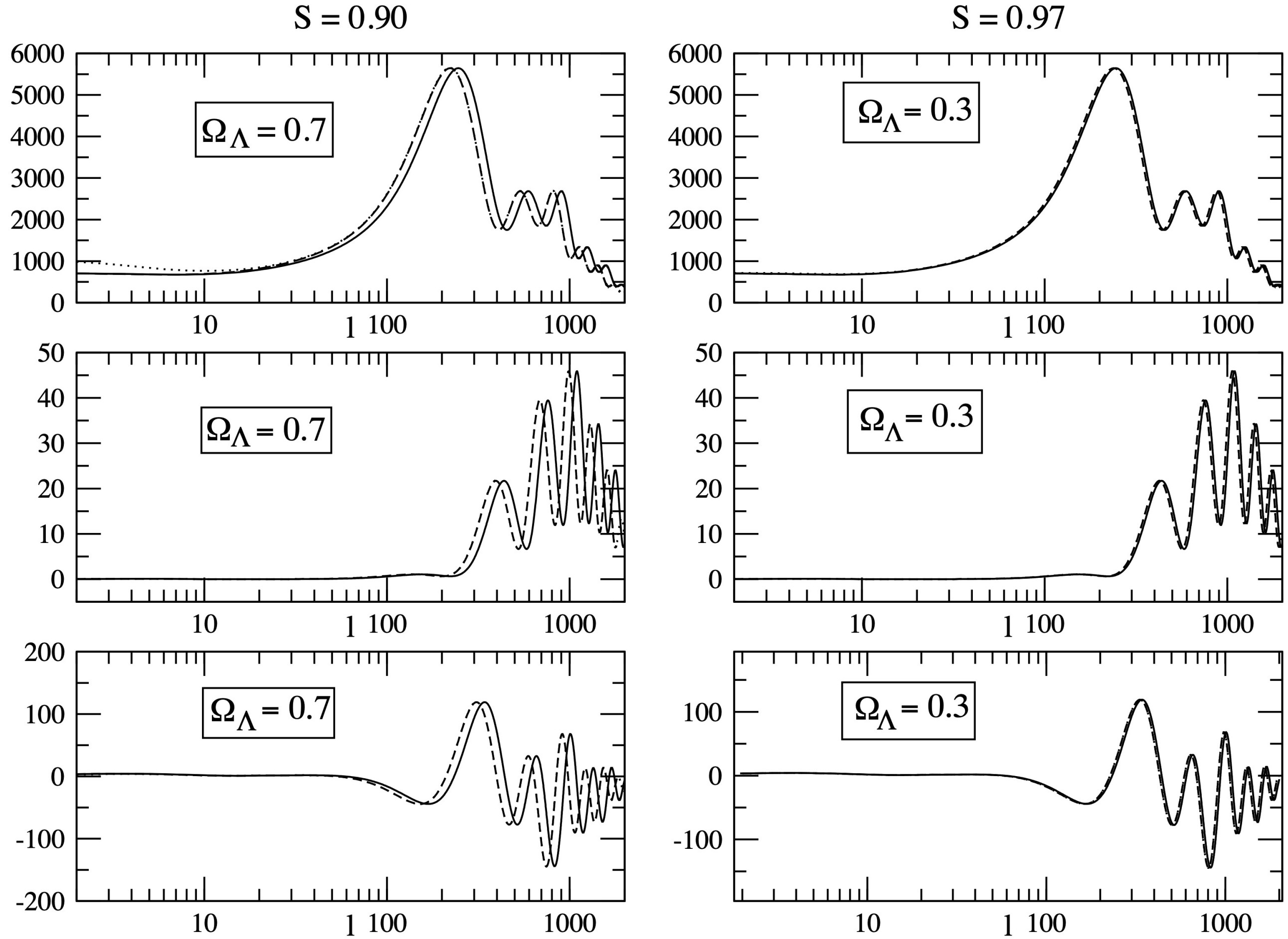
So *current data show that* LTB models on Gpc-scales *must be* **close to FL models** *with* $\Lambda \neq 0$.


Confidence interval (68%)
Mean density (68%)
$\rho(t_0, r)/\rho_{bg}$
1.14
1.00
0.85
0
1
2
3
Radial coordinate r [Gpc]

Their analysis is, as they admit, limited since linear perturbation theory for a LTB background is very complicated and still not sufficiently developed. Planck temperature power spectrum (2019):



CMB-spectra (TT, EE, TE) for $\Omega_\Lambda \neq 0$, $\Omega_K = 0$, and scaled:



Basic equations of LTB model

recall FL metric:

$$ds^2 = -dt^2 + \frac{a^2(t)}{1-kr^2}dr^2 + a^2(t)r^2 d\Omega^2, \ k = 0, \pm 1;$$

$a(t)$: scale factor; rewrite this by using $R(t,r) := a(t)r$ as

$$ds^2 = -dt^2 + \frac{(R')^2}{1-kr^2}dr^2 + R^2 d\Omega^2.$$

Lemaître metric:

$$\boxed{ds^2 = -dt^2 + \frac{(R')^2}{1-k(r)}dr^2 + R^2 d\Omega^2,}$$

dynamical variable: $R(t,r)$.

for dust (dark matter):

$$T^{\mu\nu} = \rho u^\mu u^\nu.$$

FL: dynamical law (Friedmann eq.): introduce the Hubble parameter: $H(t) := \frac{\dot{a}}{a} = \frac{\dot{R}'}{R}$, and the density parameters

$$\Omega_M = \rho_M(t_0)/\rho_{crit},\ \Omega_\Lambda = \rho_\Lambda/\rho_{crit},\ \rho_{crit} := \frac{3H_0^2}{8\pi G},\ \rho_\Lambda := \frac{\Lambda}{8\pi G},$$

and

$$\Omega_K := 1 - \Omega_M - \Omega_\Lambda.$$

Then the Friedmann equation can be written as

$$\boxed{H^2 = H_0^2\Big[\Omega_M\Big(\frac{R_0}{R}\Big)^3 + \Omega_\Lambda + \Omega_K\Big(\frac{R_0}{R}\Big)^2\Big]}$$

where $R_0(r) := R(t_0, r)$. This **remains in the LTB model** (Hamiltonian constraint), but

$$H(t) \to H(t,r),\ H_0 \to H_0(r) = \frac{\dot{R}(t_0,r)}{R(t_0,r)},\ \Omega_M \to \Omega_M(r);$$

local spatial curvature:

$$k(r) = -H_0^2 \Omega_K R_0^2.$$

free function: $\rho(t_0, r)$ (gauge choice: $R(t_0, r) = r$).
$\to$ luminosity-redshift relation (SNe Ia), etc.

(**FL**-limit: $R(t,r) \to a(t)r$)

$$t = \frac{1}{H_0(r)} \int_0^{R/R_0} \frac{dx}{\sqrt{\Omega_M(r)x^{-1} + \Omega_\Lambda(r)x^2 + \Omega_K(r)}}.$$

generalizes the basic equation (11) in Lemaître (1927)!

Luminosity distance:

$$D_L(z) = (1+z)^2 R(t(z), r(z)),$$

where $t(z)\, r(z)$ satisfy two coupled differential equations, that reduce to well-known ones for FL-models. All quantities determined by $\rho_0(r) := \rho(t_0, r)$, H_0, Λ.

Tension between local and global H_0 ?:

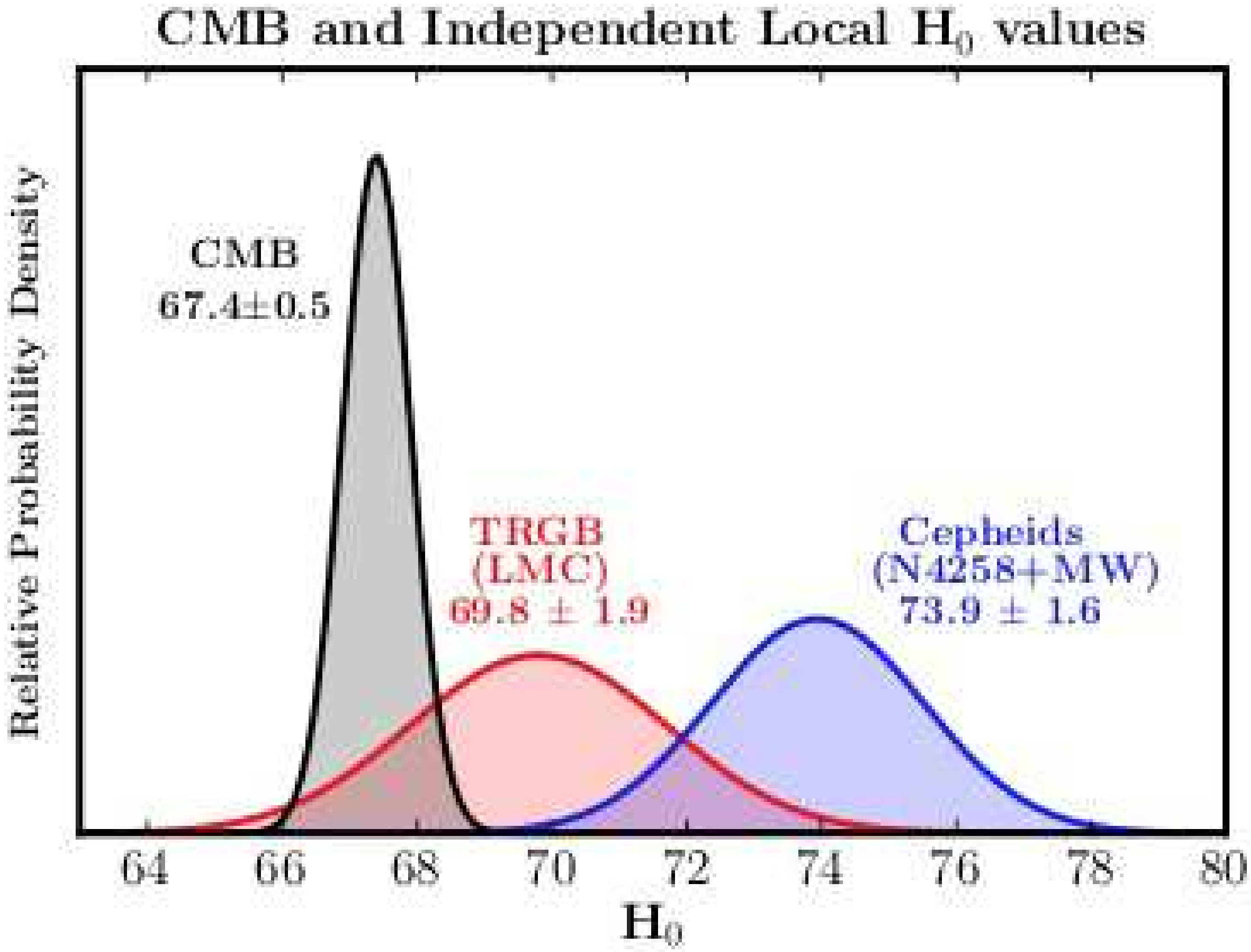
• With Cepheids and SNe Ia $H_0 = 73.24 \pm 1.74$ km s^{-1} Mpc^{-1};

• Planck for ΛCDM model, using the power spectra of CDM: $H_0 = 67.8 \pm 0.9$ km s^{-1} Mpc^{-1};

3.6σ difference; $\rightarrow$ lots of speculations, role of inhomogeneities, etc ??

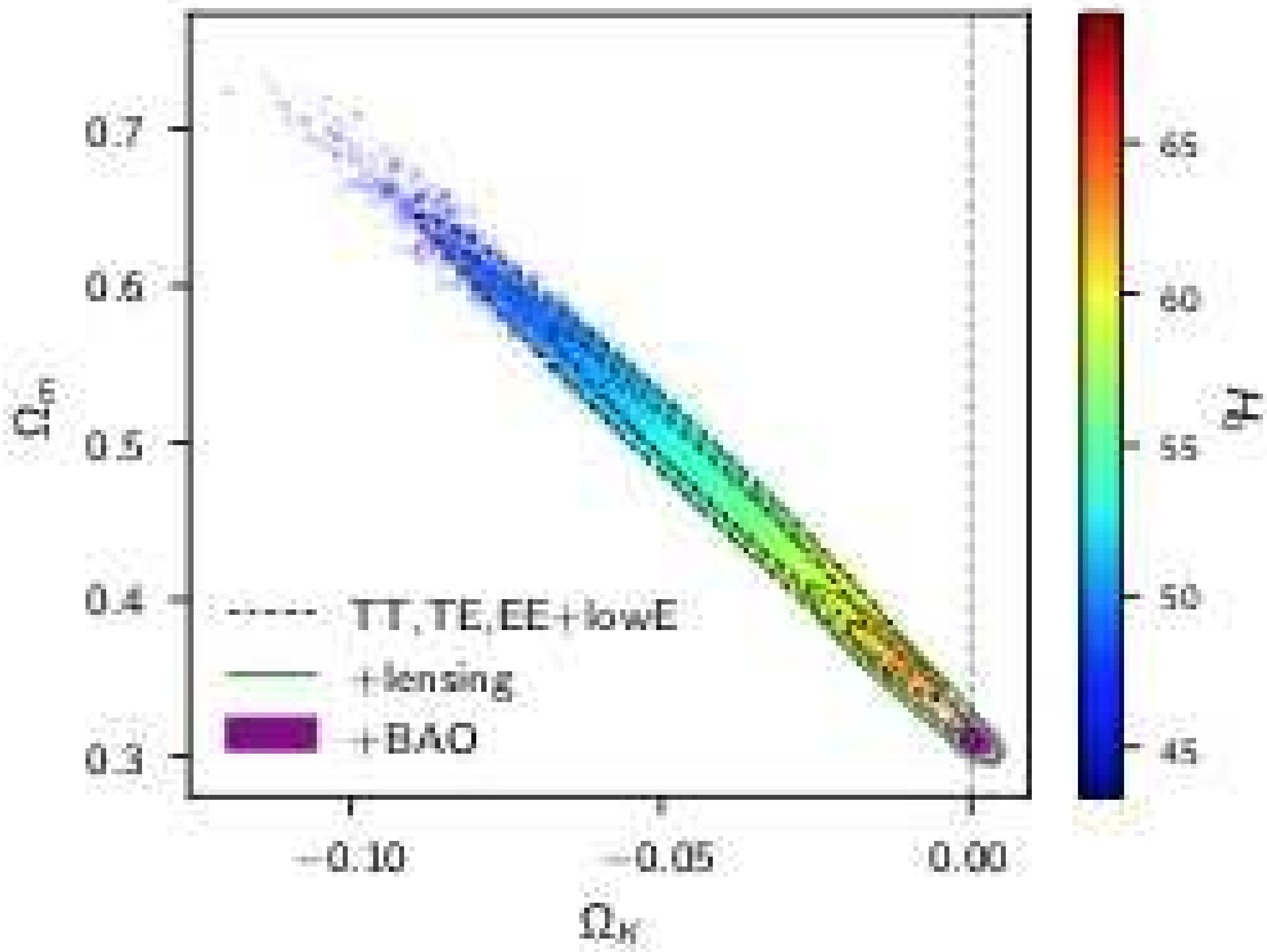
New result (last summer): W. L. Freedman et al., using the Tip of the Red Giant Branch: $H_0 = 69.8 \pm 0.8$ km s^{-1} Mpc^{-1} agrees with the global Planck value at the 1.2σ level, and is 1.75σ below the previous local one.

Exist now cosmological simulations with numerical relativity (R.D. et al.)


CMB and Independent Local H0 values
Relative Probability Density
CMB
67.4±0.5
TRGB
(LMC)
69.8 ± 1.9
Cepheids
(N4258+MW)
73.9 ± 1.6
64
66
68
70
72
74
76
78
80
H0

Lemaître (1931), "The beginning of the world from the point of view of quantum theory":

"The evolution of the world can be compared to a display of firework that has just ended: some few red wisps, ashes and smoke. Standing on a cooled cinder, we see the slow fading of the suns, and we try to recall the vanishing brilliance of the worlds".


0.7
0.6
0.5
0.4
0.3
Ω_m
−0.10
−0.05
0.00
Ω_K
TT,TE,EE+lowE
+lensing
+BAO
45
50
55
60
65
H_0