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# History and Philosophy of Physics (31)

## Wolfgang Pauli and the discovery of his Exclusion Principle hundred years ago

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## Wolfgang Pauli and the discovery of his Exclusion Principle hundred years ago

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### 1 Introduction

In this article I shall sketch how Pauli arrived at the exclusion principle hundred years ago. At the time – before the advent of the new quantum mechanics – the exclusion principle was not at all on the horizon, because of two basic difficulties: (1) There were no general rules to translate a classical mechanical model into a coherent quantum theory, and (2) the spin degree of freedom was unknown. It is very impressive indeed how Pauli arrived at his principle on the basis of the fragile Bohr-Sommerfeld theory and the known spectroscopic material. The Pauli principle was not immediately accepted, although it explained many facts of atomic physics. In particular, Heisenberg's reaction was initially very critical, as I will document later. My historical discussion will end with Ehrenfest's opening laudation [1] when Pauli received the Lorentz medal in 1931. This concluded with the words: "*You must admit, Pauli, that if you would only partially repeal your prohibitions, you could relieve many of our practical worries, for example the traffic problem on our streets.*" According to Ehrenfest's assistant Casimir who was in the audience, Ehrenfest improvised something like this: "*and you might also considerably reduce the expenditure for a beautiful, new, formal black suit*" (quoted in [2], p. 258).

Let me begin with a few biographical remarks. Pauli was born in 1900, the year of Planck's great discovery. During the high school years Wolfgang developed into an infant prodigy familiar with the mathematics and physics of his day.

### 2 Pauli's Student Time in Munich

Pauli's scientific career started when he went to Munich in autumn 1918 to study theoretical physics with Arnold Sommerfeld, who had created a "nursery of theoretical physics". Just before he left Vienna on 22 September he had submitted his first published paper, devoted to the energy components of the gravitational field in general relativity. As a 19-year-old student he then wrote two papers about the recent brilliant unification attempt of Hermann Weyl (which can be considered in many ways as the origin of modern gauge theories). In one of them he computed the perihelion motion of Mercury and the light deflection for a field action which was then preferred by Weyl. From these first papers it became obvious that Pauli mastered the new field completely.

Sommerfeld immediately recognized the extraordinary talent of Pauli and asked him to write a chapter on relativity in *Encyklopädie der mathematischen Wissenschaften*. Pauli was in his third term when he began to write this article. Within less than one year he finished this demanding job, beside his other studies at the university. With this article [3] of 237 pages and almost 400 digested references Pauli established himself as a scientist of rare depth and surpassing synthetic and critical abilities. Einstein's reaction was very positive:

> *"One wonders what to admire most, the psychological understanding for the development of ideas, the sureness of mathematical deduction, the profound physical insight, the capacity for lucid, systematic presentation, the knowledge of the literature, the complete treatment of the subject matter or the sureness of critical appraisal."*

Hermann Weyl was also astonished. Already on 10 May 1919, he wrote to Pauli from Zürich:

> *"I am extremely pleased to be able to welcome you as a collaborator. However, it is almost inconceivable to me how you could possibly have succeeded at so young an age to get hold of all the means of knowledge and to acquire the liberty of thought that is needed to assimilate the theory of relativity."*

Pauli studied at the University of Munich for six semesters. At the time when his Encyclopedia article appeared, he obtained his doctorate with a dissertation on the hydrogen molecule ion $H_2^+$ in the old Bohr-Sommerfeld theory. In it the limitations of the old quantum theory showed up.

In the winter semester of 1921/22 Pauli was Max Born's assistant in Göttingen. During this time the two collaborated on the systematic application of astronomical perturbation theory to atomic physics. Already on 29 November 1921, Born wrote to Einstein: "*Little Pauli is very stimulating: I will never have again such a good assistant.*" Well, Pauli's successor was Werner Heisenberg.

### 3 Discovery of the Exclusion Principle

Pauli's next stages were in Hamburg and Copenhagen. His work during these crucial years culminated with the proposal of his exclusion principle in December 1924. This was Pauli's most important contribution to physics, for which he received a belated Nobel Prize in 1945. Since this was made before the advent of the new quantum mechanics, I ask you to forget for a while what you know about quantum mechanics.

The discovery story begins in fall 1922 in Copenhagen when Pauli began to concentrate his efforts on the problem of the anomalous Zeeman effect. He later recalled: '*A colleague who met me strolling rather aimlessly in the beautiful streets of Copenhagen said to me in a friendly manner, "You look very unhappy"; whereupon I answered fiercely, "How can one look happy when he is thinking about the anomalous Zeeman effect?" '.*

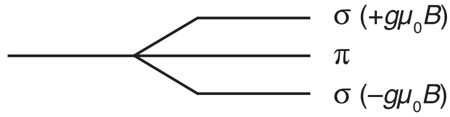
In a Princeton address in 1946 [4], Pauli tells us how he felt about the anomalous Zeeman effect in his early days:

*"The anomalous type of splitting was on the one hand especially fruitful because it exhibited beautiful and simple laws, but on the other hand it was hardly understandable, since very general assumptions concerning the electron, using classical theory as well as quantum theory, always led to a simple triplet. A closer investigation of this problem left me with the feeling that it was even more unapproachable (...). I could not find a satisfactory solution at that time, but succeeded, however, in generalizing Landé's analysis for the simpler case (in many respects) of very strong magnetic fields. This early work was of decisive importance for the finding of the exclusion principle."*

### What was known at the time when Pauli began with his work

I would like to show you now in some detail what Pauli did in his first step [5]. In doing this, I use 'modern' (post-quantum mechanics) notations and first summarize the state of knowledge at the time when Pauli did his work.

- The energy levels of an atom determine the spectrum by *Bohr's rule*:
  $$E_2 - E_1 = h\nu.$$
- In spectroscopy some *quantum numbers* were already associated to energy levels, namely [1]:
  - ▶ $L$ [$= k - 1$], $L = 0, 1, 2, 3,...$ (S, P, D, F,...), our present day orbital angular momentum.
  - ▶ $S$ [$= i - ½$]: Each term belongs to a singlet or multiplet system, characterized by a *maximal* multiplicity $2S + 1$ ($S = 0, ½ , 1, ...$), reached with increasing $L$. $S$ is our present day spin quantum number.
  - ▶ The various terms of a multiplet, having the *same* $L$ and $S$, are distinguished by a quantum number $J$ [Sommerfeld's $j$], which takes the values:
    $J = L + S, L + S - 1,... L - S$ for $L \geq S$,
    $J = S + L, S + L - 1,... S - L$ for $L < S$.
    $J$ is our present day total angular momentum. The maximal multiplicity $2S + 1$ is reached for $L \geq S$.
- One knew the following *selection rules* (valid in most cases):
  $L \to L \pm 1$,
  $S \to S$,
  $J \to J + 1, J, J - 1$ ($0 \to 0$ forbidden).
- For a given atomic number $Z$ ($Z - p$ if the atom is ionized $p$ times) the following holds:
  $Z$ even $\to$ $S$, $J$: integer,
  $Z$ odd $\to$ $S$, $J$: halfinteger.
- *Splitting in a magnetic field*:
  - ▶ Each term splits into $2J + 1$ terms, distinguished by a quantum number $M$ taking the values $M = J, J - 1, ..., -J$.
  - ▶ *Landé*: If the field is *weak*, the terms are equidistant and their deviation from the unperturbed term is $\Delta E_M = M \cdot g(\mu_0 B)$, where $\mu_0 = e\hbar/2mc$ is the Bohr magneton (introduced by Pauli in 1920) and $g$ is Landé's $g$ factor:
    $$g = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)} \;;$$
    (extracted from empirical data; not understandable on classical grounds.)
  - ▶ *Selection rules* for Zeeman transitions:
    $M \to M \pm 1$ ($\sigma$–component),
    $M \to M$ ($\pi$–component).
    If the $g$ factors for the initial and final states are the same, we have the following Lorentz triplet (*normal* Zeeman effect):




### Step 1: Zeeman effect for strong fields and Pauli's sum rule

Pauli accepts these *empirical rules* as established, and proceeds to investigate the spectroscopic material for *strong* fields (Paschen-Back case). In a table he gives the energy splitting's $\Delta E$ as multiples of $\mu_0 B$ and describes the result as follows:

If two quantum numbers $M_L$, $M_S$ [$= m_1, \mu$] are introduced, whose sum is equal to $M$,
$$M = M_L + M_S,$$
and which take the values
$$M_L = L, L{-}1,..., -L,$$
$$M_S = \begin{cases} \pm\frac{1}{2} & \text{for doublets (alkali atoms)} \\ 0, \pm 1 & \text{for triplets (alkaline earths)} \end{cases},$$
then the following simple formula holds for strong fields:
$$\Delta E/\mu_0 B = M_L + 2M_S = M + M_S.$$

Pauli *generalizes* this at once to arbitrary multiplets, assuming that the same formula holds, but that $M_S$ takes the values
$S, S - 1, ..., -S$.

This generalization was at the time not experimentally tested.

The selection rule for $M_S$ is: $M_S \to M_S$, hence the Zeeman effect is *normal for strong fields*. Thus the situation is simpler in this case.

As the main point of the paper Pauli postulates a remarkable formal rule which allows him to derive Landé's whole set of $g$ factors. Pauli's sum rule reads:

*"The sum of the energies of all states of a multiplet belonging to given values of M and L remains a linear function of B, when we pass from weak to strong fields."*

(In quantum mechanics this rule follows immediately [2])

**Special cases:**

1) $M = J = L + S$. Then there is only *one* state, whose energy must be linear in $B$.
2) If $M$ is chosen such that there are $2S + 1$ states (the maximal possible number) then the arithmetic mean of

1 In square brackets I give the historical notation.

2 The sum in Pauli's rule is the trace of $<H_B>$, $H_B = \mu_0 B(J_3 + S_3)$, where $<H_B>$ is the perturbation matrix for fixed $M$. This trace is obviously linear in $B$. For this we have from the anomalous Zeeman-effect and the Paschen-Back-effect two alternative expressions. Their equality leads to the following equation
$$\sum_{M_L, M_S}^{M_L+M_S=M} \frac{M_L + 2M_S}{M_L + M_S} = \sum_J g(J; L, S) \;.$$

their energies $\Delta E_M$ in strong magnetic fields is $M\mu_0 B$. Hence Pauli's sum rule, which later became known as *Pauli's g-permanence rule*, implies that the mean of all factors is equal to 1, for all $L \geq S$.

Pauli now shows – and he puts most weight to this – that all factors *g* can uniquely be calculated from the energies for strong fields. Pauli verified this only numerically, but claimed that "it is possible to carry through the calculation also algebraically instead of numerically". (This is shown in the biography [2] on Pauli by Charles P. Enz, on p. 164.)

Pauli was very unhappy when he wrote this paper, which only later turned out to be important. In several letters he laments about his 'unfortunate work on the anomalous Zeeman effect'. To Sommerfeld he wrote [3]:

*"I have long vexed myself with the anomalous Zeeman effect and often lost my way. I considered and discarded untold assumptions. But it just wouldn't ever work out! In this I have miserably failed for once up to now! For a time I was quite desperate ... I have written all of this with a tear in the corner of my eyes and am anything but delighted."*

In the final section of his paper he expresses very clearly why he believes that the presently known principles of quantum theory will not lead to an understanding of the anomalous Zeeman effect. Since I find it very difficult to preserve the characteristic style of Pauli's writing I quote only the German original:

*"Eine befriedigende modellmässige Deutung der dargelegten Gesetzmässigkeiten, insbesondere der in diesem Pararaphen besprochenen formalen Regel ist uns nicht gelungen. Wie schon in der Einleitung erwähnt, dürfte eine solche Deutung auf Grund der bisher bekannten Prinzipien der Quantentheorie kaum möglich sein. Einerseits zeigt das Versagen des Larmorschen Theorems, dass die Beziehung zwischen dem mechanischen und dem magnetischen Moment eines Atoms nicht von so einfacher Art ist wie es die klassische Theorie fordert, indem das Biot-Savartsche Gesetz verlassen oder der mechanische Begriff des Impulsmomentes modifiziert werden muss. Anderseits bedeutet das Auftreten von halbzahligen Werten von m und j bereits eine grundsätzliche Durchbrechung des Rahmens der Quantentheorie der mehrfach periodischen Systeme."*

After his return to Hamburg Pauli began to think about the closing of electronic shells. He was convinced that there must be a closer relation of this problem to the theory of multiplet structure. In his Nobel Prize lecture he writes:

*"I therefore tried to examine again critically the simplest case, the doublet structure of the alkali spectra. According to the point of view then orthodox, which was also taken over by Bohr in his lectures in Göttingen, a non-vanishing angular momentum of the atomic core was supposed to be the cause of this doublet structure."*

3 "*Ich habe mich sehr lange mit dem anomalen Zeemaneffekt geplagt, wobei ich oft auf Irrwege geriet und eine Unzahl von Annahmen prüfte und dann wieder verwarf. Aber es wollte und wollte nicht stimmen! Dies ist mir bis jetzt einmal gründlich danebengegangen! Eine Zeit lang war ich ganz verzweifelt ... ich habe das Ganze mit einer Träne im Augenwinkel geschrieben und habe davon wenig Freude.*"

In his next paper [6] Pauli rejected this 'orthodox' point of view, and introduced instead a classically non-describable two-valuedness of the electron, now called the spin.

***Step 2: Two-valuedness of the electron***

Let me show you in some detail how he arrived at this fundamental conclusion. First, he calculates the relativistic corrections upon the magnetic moment and the orbital angular momentum of electrons in the K-shell. For the ratio of the two he finds with simple classical arguments

$$\frac{|\vec{M}|}{|\vec{L}|} = \frac{e}{2mc}\left\langle\left(1-\frac{v^2}{c^2}\right)^{1/2}\right\rangle_{\text{time}} .$$

According to the virial theorem the time average on the right must be equal to the total energy of the electron in units of $mc^2$. For the latter Pauli uses Sommerfeld's relativistic formula and finds for the K-shell ($L = 0$, $n = 1$) the value $(1-\alpha^2 Z^2)^{1/2}$ for the relativistic correction factor ($\simeq 1-\frac{1}{2}\alpha^2 Z^2/n^2$ for an arbitrary $n$).

Adopting the 'orthodox' point of view, Pauli now calculates the relativistic correction on the anomalous Zeeman effect, using his earlier results – in particular his sum rule. I do not have to tell you this in detail, because it turns out that this influence on the g-factors is *not* compatible with experience. The empirical factors *g* are rational numbers depending only on the quantum numbers of the term. The result is summarized by Pauli as follows:

*"In order to explain the observed factors g by means of an angular momentum of closed shells, such as the K-shell of the alkali atoms, one would have to assume a doubling of the ratio of magnetic to mechanical momentum for electrons in the shell, and also a compensation of the classically computed relativistic effect of velocity,"*

Pauli rejects this logical possibility. Instead he assumes that closed shells have no angular momentum and no magnetic moment. This implies that in the case of alkali atoms the angular momentum of the atom and its change of energy in a magnetic field are ***due to the valence electron only***. In Pauli's words:

*"Insbesondere werden bei den Alkalien die Impulswerte des Atoms und seine Energieänderungen in einem äusseren Feld im wesentlichen als alleinige Wirkung des Leuchtelektrons angesehen, das auch als Sitz der magneto-mechanischen Anomalie betrachtet wird."*

So far Pauli had only made a critical analysis of an existing hypothesis, but now comes a big jump when he writes:

*"According to this point of view the doublet structure of alkali spectra as well as the deviation from Larmor's theorem is due to a particular two-valuedness of the quantum theoretical properties of the electron, which cannot be described from the classical point of view."*

Since Pauli does not explain these prophetic words any further in this second paper, it may be helpful if I add a few remarks. For *strong* fields he had the formulae

$$M = M_L + M_S,\ \Delta E/\mu_0 B = M_L + 2M_S.$$

Pauli follows Sommerfeld and interprets *M* in his next paper as the total angular momentum in the direction of the field.

(Sommerfeld also introduced the quantum number $J$.) For alkali atoms the closed shells do not contribute to $M$ nor to the magnetic moment. Hence,

$$M = m_\ell + m_s,\ \Delta E/\mu_0 B = m_l + 2m_s,$$

where $m_\ell$, $m_s$ are the values of $M_L$ and $M_S$ for the single valence electron. The integer number $m_\ell$ may be interpreted classically as the orbital angular momentum in the direction of the field. Therefore, $m_s$ is an *intrinsic* contribution of the electron to the total angular momentum $M$ in the direction of the field which must be added to $m_\ell$. We have already seen that for the alkali doublets $m_s$ takes the values ±½.

Since $m_\ell$ is an integer it follows that $M$ is a half-integer, and since $J$ of a multiplet is defined to be the maximal value of $M$, we have for the *two terms of an alkali doublet*

$$J = L \pm ½.$$

Thus, the two-valuedness of $J$, which is responsible for the doublet splitting, is a direct consequence of the two-valuedness of $m_s$. This explains the first part of Pauli's key sentence:

> *"...the doublet structure of alkali spectra (...) is due to a particular two-valuedness (...) of the electron."*

What exactly did Pauli mean concerning "the deviation of Larmor's theorem"?

In the paper of Pauli I have just discussed he uses the well-known formula for the energy of an atom in a magnetic field

$$\Delta E = -\vec{M} \cdot \vec{B}, \qquad \vec{M}\text{: magn. moment.}$$

If we compare this with his expression for strong fields, we see that an atom behaves in a strong field like a magnet having a magnetic moment $\mu_0(M_L + 2M_S)$ in the direction of the field. For a single valence electron this is equal to $\mu_0(m_\ell + 2m_s)$.

So far strong fields have been assumed. But if we consider an $S$ state of an alkali atom, we have $M_L = 0$, $M = M_S = m_s$, and now the formula $\Delta E/\mu_0 B = 2m_s$ holds – by Pauli's sum rule – for *weak* fields too. This means: For $S$ states the magnetic moment of alkali atoms is equal to $\underline{2m_s\mu_0}$ (the famous $g = 2$). According to Pauli, this magnetic moment is *entirely due to the valence electron*.

Pauli did not attempt to give a meaning to the fourth degree of freedom in terms of a model. In his Nobel Prize lecture [7] he said about this:

> *"The gap was filled by Uhlenbeck and Goudsmit's idea of electron spin, which made it possible to understand the anomalous Zeeman effect. (...) Although at first I strongly doubted the correctness of this idea because of its classical mechanical character, I was finally converted to it by Thomas' calculations on the magnitude of doublet splitting."*

***Step 3: The exclusion principle***

In his decisive third paper [8] Pauli first summarizes his previous results for alkali metals. For these the quantum numbers $L$, $J$, $M$ of the atom coincide with those of the valence electron for which we use the modern notation $\ell$, $j$, $m_j$. (Pauli's notation is: $k_1 = \ell + 1$, $k_2 = j + ½$, $m_1 = m_j$.) Beside these there is, of course, also the principle quantum number $n$. As already explained, the number $j$ is equal to $\ell \pm ½$. Pauli emphasizes:

> *"The number of states in a magnetic field for given $\ell$ and $j$ is $2j + 1$, the number of these states for both doublets with a given $\ell$ taken together is $\underline{2(2\ell + 1)}$."*

For the case of *strong* fields, Pauli adds, one can use instead of $j$ the quantum number $m_2 := m_j \pm ½$ ($= m_j + m_s$), which directly gives the component of the magnetic moment parallel to the field. (We would use for the Paschen-Back region the four quantum numbers $n$, $\ell$, $m_\ell$, $m_s$.)

Next, Pauli extends the "formal classification of the valence electron by the four quantum numbers $n$, $\ell$, $j$, $m_j$ to *complicated atoms*". This is performed with the help of Bohr's *principle of permanence* (*Aufbauprinzip*), which says: If, to a partially ionized atom, one (or more) electron is added, the quantum numbers of the electrons already bound remain the same as in the ionized atom. Pauli shows that in simple as well as in more complicated cases the application of this principle gives just the right variety of terms for the atom.

For Pauli's further line of thought the formulae for the Zeeman effect in *strong* fields are again essential. First, the principle of permanence implies that one can associate quantum numbers $m_j$ for the individual electrons, the sum of which is the total angular momentum of the atom in the direction of the field:

$$M = \sum m_j\ .$$

By the same rule, the magnetic moment $(M + M_S)\mu_0$ is also equal to the sum of the moments $m_2\mu_0$ of all the electrons, i.e.,

$$M_2 := M_L + 2M_S = M + M_S = \sum m_2\ .$$

In the sums $m_j$ and $m_2$ have to assume *independently* all values which belong to the quantum numbers $j$, $\ell$. Pauli checked (for instance for neon) that this gives the correct results for the Zeeman terms.

This result, he says, suggests the following hypothesis: "*Every electron in the atom can be characterized by its principle quantum number n and **three additional quantum numbers** $\ell$, $j$, $m_j$*". As for the alkali spectra, $j$ is always equal to $\ell$ ± ½. For strong fields the quantum number $m_2 = m_j \pm ½$ is used instead of $j$.

It must be emphasized that Pauli had to assume a magnetic field so strong that every electron has, *independently* of the others, a definite mechanical angular momentum $m_j$ and a magnetic moment $m_2$ (in units of $\mu_0$), but he notes that for thermodynamic reasons (invariance of the statistical weights under adiabatic transformations of the system) the number of states in weak fields must be the same as in strong fields. In an article of van der Waerden [9], I have made heavy use of in this section, this is commented as follows:

> *"It is clear that the definition of these quantum numbers presented great difficulties at a time when quantum mechanics did not exist and the types of motion of the electron had to be described by inadequate classical models. (...) We have to admire Pauli's courage and persistence*

*in developing the logical consequences of his hypothesis. The subsequent development of quantum mechanics led to a complete justification of every one of his assumptions."*

Next, Pauli considers the case of *equivalent electrons*. First of all he notes that in this case some combinations of quantum numbers do *not occur in nature*. For instance, if two valence electrons are in *s* states belonging to different values of *n*, we observe a singlet *S* term and a triplet *S* term. If, however, both electrons have the same *n*, *only the singlet term occurs*. For Pauli the question arises, which quantum theoretical rules govern this behavior of the terms.

This reduction of terms, Pauli says, is closely connected with the phenomenon of closed shells. About this E. C. Stoner [10] had recently made a new proposal which deviated from Bohr's theory of the periodic system. For example, Bohr had divided the 8 electrons of the *L*-shell into two subgroups of 4 electrons. Stoner, on the other hand, proposed to divide the electrons into a subgroup of 2 electrons having $\ell = 0$, and a subgroup of 6 electrons with $\ell = 1$. Generally, for any closed shell and every value of $\ell < n$, Stoner associated a *subgroup of* $2(2\ell + 1)$ *electrons*.

Even more important was Stoner's remark that the same number $2(2\ell + 1)$ is also equal to the number of states of an *alkali atom in a magnetic field* belonging to the same value of $\ell$ and to a given principle quantum number of the valence electron. This remark of Stoner gave Pauli the clue to his exclusion principle. He explains the fact that there are exactly $2(2\ell + 1)$ electrons in every subgroup of a closed shell by assuming that every state, characterized by the quantum numbers $(n, \ell, j, m_j)$, is **occupied by just one electron**. Then we have for a given $n$ and $\ell > 0$ just the two possibilities $j = \ell \pm ½$ with $2j + 1$values for $m_j$, giving together $2(2\ell + 1)$ electrons.

In Pauli's words of his Nobel Prize lecture [7]:

*"The complicated numbers of electrons in closed subgroups reduce to the simple number* ***one*** *if the division of the groups by giving the values of the 4 quantum numbers of an electron is carried so far that every degeneracy is removed. A single electron already occupies an entirely non-degenerate energy level."*

In his original paper Pauli enunciates his principle as follows:

*"There can* ***never be two or more equivalent electrons*** *in an atom, for which in strong fields the values of all quantum numbers* $n, \ell, j, m_j$ *are the same. If an electron is present in the atom, for which these quantum numbers have definite values, this state is 'occupied'."*

From this Pauli deduces the numbers **2**, **8**, **18**, **32**,**...** of electrons in closed shells, and the **reduction of terms** for equivalent electrons. Several further applications are always in accordance with experience.

At the end of his paper Pauli expresses the hope that a deeper understanding of quantum mechanics might lead to a derivation of the exclusion principle from more fundamental hypothesis. To some extent this hope was fulfilled in the framework of relativistic quantum field theory. Pauli's key role in establishing the *spin-statistic theorem* is well-known (see, e.g., [11]).

Initially Pauli was not sure to what extent his exclusion principle would hold good. In a letter to Bohr of 12 December 1924 Pauli writes '*The conception, from which I start, is certainly nonsense. (...) However, I believe that what I am doing here is no greater nonsense than the hitherto existing interpretation of the complex structure. My nonsense is conjugate to the hitherto customary one.*' The exclusion principle was not immediately accepted, although it explained many facts of atomic physics. A few days after the letter to Bohr, Heisenberg wrote to Pauli on a postcard: '*Today I have read your new work, and it is certain that I am the one who rejoices most about it, not only because you push the swindle to an unimagined, giddy height (by introducing individual electrons with 4 degrees of freedom) and thereby have broken all hitherto existing records of which you have insulted me. (...).*'.

For the letters of Pauli on the exclusion principle, and the reactions of his influential colleagues, I refer to Vol. 1 of the *Pauli Correspondence*, edited by Karl von Meyenn [12]. Some passages are translated into English in the scientific biography [2] by Charles Enz.

## 4 Exclusion principle and the new quantum mechanics

On 26 August 1926, Dirac's paper containing the Fermi-Dirac distribution was communicated by R. Fowler to the Royal Society. This work was the basis of Fowler's *theory of white dwarfs*. I find it remarkable that the quantum statistics of identical spin-½ particles found its first application in astrophysics. Pauli's exclusion principle was independently applied to *statistical thermodynamics* by Fermi [4].

In the same year 1926, Pauli simplified Fermi's calculations, introducing the grand canonical ensemble into quantum statistics. As an application he studied the behavior of a gas in a magnetic field (paramagnetism).

Heisenberg and Dirac were the first who interpreted the exclusion principle in the context of Schrödinger's wave mechanics for systems of more than one particle. In these papers it was not yet clear how the spin had to be described in wave mechanics. (Heisenberg speaks of spin coordinates, but he does not say clearly what he means by this.) The definite formulation was soon provided by Pauli in a beautiful paper [14], in which he introduced his famous spin matrices.

At this point the foundations of non-relativistic quantum mechanics had been completed in definite form. For a lively discussion of the role of the exclusion principle in physics and chemistry from this foundational period, I refer once more to the address [1] of Ehrenfest.

4 According to Max Born, Pascual Jordan was actually the first who discovered what came to be known as the Fermi-Dirac statistics. Unfortunately, Born, who was editor of the *Zeitschrift für Physik*, put Jordans paper into his suitcase when he went for half a year to America in December of 1925, and forgot about it. For further details on this, I refer to the interesting article [13] by E. L. Schucking.

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